DOCUMENT RESUME

ED 367 531 SE 053 821

TITLE	Arizona Essential Skills for Mathematics. Reformatted Edition.
INSTITUTION	Arizona State Dept. of Education, Phoenix.
PUB DATE	Sep 92
NOTE	235p.; For earlier edition, see ED 325 364.
AVAILABLE FROM	Arizona State Dept. of Education, 1535 W. Jefferson, Phoenix, AZ 85007.
PUB TYPE	Guides - Classroom Use - Teaching Guides (For Teacher) (052)
EDRS PRICE	MF01/PC10 Plus Postage.
DESCRIPTORS	Algebra; Calculus; *Curriculum Development; *Educational Assessment; Educational Objectives; Elementary Secondary Education; Fractions; Geometry; Mathematical Applications; Mathematical Concepts; Mathematical Logic; *Mathematics Curriculum; Mathematics Education; *Mathematics Skills; Measurement; Numeracy; Probability; Problem Solving; *State Standards; Statistics; *Student Evaluation; Trigonometry
IDENTIFIERS	*Arizona; Mathematical Communication; Mathematical Thinking; NCTM Curriculum and Evaluation Standards

ABSTRACT

This document ties the essential skills needed in mathematics to the National Council of Teachers of Mathematics' (NCTM) Curriculum and Evaluation Standards to help facilitate future curriculum development. The overall goal for the 21st century is to make mathematical power a reality for all students. The rationale for this document can be found in three goals: (1) to restructure mathematics curricula and teaching strategies to change mathematics from a static discipline to a dynamic process; (2) to set high standards of numeracy for all students; and (3) to integrate student assessment with the learning process. This document divides essential mathematical skills into three grade levels: K-3, 4-8, and 9-12. Each section presents the essential skills needed at each level in matrix form showing the direct relationship between the content skills and the NCTM Standards. Each section also supplies a list of student outcomes for each content area, as well as examples of indicators. A direct assessment program which can be used in meeting goal 3 is presented. Contains 22 references. (RLB)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED 367 531

ARIZONA ESSENTIAL SKILLS FOR MATHEMATICS

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Linda Edgington

Librarian II

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE 053 821

Arizona Department of Education
C. Diane Bishop, Superintendent
September 1992

Reformatted Edition

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

X☐ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

BEST COPY AVAILABLE

STATE BOARD OF EDUCATION

Claudeen Bates Arthur

Kenneth Bennett

Honorable C. Diane Bishop

Dr. John Hosmer

Dr. Eugene Hughes

Dr. Raymond Kellis

Honorable David Silva

James Ullman

Dr. Morrison Warren

The Arizona Department of Education is a an equal opportunity employer and educational agency and affirms that it does not discriminate on the basis of race, color, national origin, age, sex, or handicapping condition.

ADE

Printed in Phoenix, Arizona by the
Arizona Department of Education
Total Copies Printed.....3000
Total Printing Cost.....$6290.00
Unit Printing Cost.....$2.10
Date of Printing.....1/93

Contents

	Page
Foreword	i
Background on the 1987 Document	iii
Introduction	1
Goals for Entering the 21st Century	2
Rationale	3
It's Time to Change	3
Let's Change with the Times	3
Making the Commitment	5
Meeting the Challenge	6
Make Mathematical Power a Reality for All Students	7
From Goals to Action: It's Time	8
GOAL 1. Restructure Mathematics Curricula and Teaching Strategies to Change Mathematics from a Static Discipline to a Dynamic Process	9
GOAL 2. Set High Standards of Numeracy for All Students	10
How This Section Is Organized	10
K-3 MATHEMATICS ESSENTIAL SKILLS	13
4-8 MATHEMATICS ESSENTIAL SKILLS	33
9-12 MATHEMATICS ESSENTIAL SKILLS	65
GOAL 3. Integrate Student Assessment with the Learning Process	98
Arizona Student Assessment Program (ASAP)	98
District Assessment Plan (DAP)	103
Performance in the Classroom	107
Scoring	112
Resources	115

Foreword

The *Arizona Essential Skills for Mathematics* was developed in 1987 by the Essential Skills Committee appointed by the state Board of Education. The document served as the framework that guided the development of the mathematics portion of the Arizona Student Assessment Program. But the 1987 document was interpreted by many districts as a list of isolated skills and was taught and assessed accordingly. The intent of the document was far more comprehensive. In 1991, a committee was appointed by the state Superintendent to reformat the Arizona Essential Skills for Mathematics in order to illustrate the intent of the original Essential Skills document.

The process used to develop the original document is shown on pages ii, iii and iv. The Essential Skills included in the 1987 document were based on the 1986 California Framework developed by the California Department of Education. The 1991 committee reformatted the document and aligned it to the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics*. The National Council of Teachers of Mathematics has given the Arizona Department of Education permission to reprint portions of the *Curriculum and Evaluation Standards for School Mathematics* in the 1992 Essential Skills document.

The 1991 Mathematics Essential Skills Committee was chaired by Linda Jaslow, mathematics specialist at the Arizona Department of Education. Members of the committee were

Micki Aley, Arizona Department of Education
Dan Benson, Phoenix Union High School District
Jim Cochrane, Marana Unified School District
Nancy Fisk, Balsz Elementary School District
Jeri Hamilton, Prescott Unified School District
Christene Meister, Tempe Elementary School District
Nancy Ostergren, Litchfield Elementary School District
Colette Pointer-Pullen, Alhambra Elementary School District
Gerri Shaft, Marana Unified School District

Background on the 1987 Document

The state Board of Education appointed the 1987 advisory committee to perform the initial tasks involved in revision and review. One-third of the Committee members were appointed as professional educators. The remaining two-thirds of the Committee were citizen members, although several had past and present experience as educators, many as former teachers or members of school boards or districts. Notwithstanding their extremely broad and diverse backgrounds, Committee members shared an intense interest in mathematics education.

The educator members of the Committee were from all sectors of education, including elementary, secondary and university teachers and administrators. A staff facilitator was appointed to the Committee by the Department of Education at the Board's request. This person was Kay Dean, mathematics specialist.

Other key resources at the Committee's disposal were mathematics curricula from several other states and some of the foreign countries which scored higher than the United States on the recent International Mathematics Test. Some of the approaches from the other states and countries were found suitable or adaptable for use in Arizona and were incorporated into the Committee's final report and the Essential Skills themselves.

The Mathematics Essential Skills Committee was chaired by James Nelson from Phoenix. Members of the Committee and communities they represented are

Chambers:	Elsie Duran	Scottsdale:	Larry Smith
Eagar:	Stan Smith	Tempe:	Dr. Ron Brown
			Jan Burkhardt
Flagstaff:	Charles Little		Joan Pfuhl
Mesa:	Bonnie Morales	Tucson:	Carol Brooks
	Dr. Steven Walters		Dale Curtis
			Ilona Gay
Phoenix:	Martha Baca		Dr. Virginia Horak
	Cindy Gilbert		
	Joy Hanley	Williams:	Dan Baertlein
	Joanne Kimura		
	Jan Miller		
	James Nelson		
	Adria Renke		
	Susan Saldaña		
	Dean Sulzer		

The Process

The Committee had its first meeting in May 1986. Subsequent to this initial meeting, the Committee met on a monthly basis through December 1986.

During the course of discussions, the Committee displayed a willingness to examine in great detail all facets of mathematics education. The Committee made a conscious decision early in the process, however, to keep the "big picture" in focus even while examining the details necessary for establishing the Essential Skills. Among the important considerations guiding the Committee were the following:

1. The overall objective of these Essential Skills is to increase student math proficiency. This objective can only be met by improving the quality of mathematics instruction.

2. The Essential Skills outlined here are intended as a flexible guide for the local districts. The Essential Skills should be adapted to local conditions so that the needs of all students in the educational community can be met.

3. The members of the Committee were uniform in their belief that districts and schools should be responsible for teaching these skills and striving to exceed them whenever possible.

4. Although the Essential Skills were designed primarily as a guide to local districts and individual schools in their development of curriculum and implementation of that curriculum, they also are intended to provide guidance to the textbook selection committee.

District Responses and Hearings

The Committee was keenly aware that the success or failure of the proposed revisions to the Essential Skills in Mathematics were contingent, in large part, on the acceptance and implementation of those skills by local districts and schools. Therefore, a draft of the proposed revisions was submitted to school districts statewide for district-level input.

Superintendents were asked to respond to the revision or designate someone else in their district to do so. The individual districts were also provided a response sheet on which they could report their impressions, and were informed of the dates, places and times of the three regional public hearings.

Response sheets were received from many districts and schools within districts. Some districts and schools made copies of the entire document and teachers were asked to respond. All responses were reviewed by Committee members. Complete data, including a narrative of comments and statistical reports, can be obtained from the Department of Education. Some districts, schools and teachers wrote separate letters or attached separate comments, and these narrative responses are also available from the Department of Education.

Hearings were scheduled in three locations across the state to provide for direct citizen and district input.

Schedule of Public Hearings

Flagstaff, Arizona

Place:	Flagstaff High School Mini-Auditorium
Date:	Tuesday, January 13, 1987
Time:	7 p.m. – 9 p.m.

Phoenix, Arizona

Place:	Arizona Department of Education 1535 West Jefferson, Room 417
Date:	Wednesday, January 14, 1987
Time:	5 p.m. – 7 p.m.

Tucson, Arizona

Place:	Board Room, Tucson Unified School District No. 1 1010 East Tenth Street
Date:	Thursday, January 15, 1987
Time:	5 p.m. – 7 p.m.

Approximately 50 people attended the hearings, and participants' comments were recorded on notes and tape. A summary of the notes taken at the hearings is available from the Department of Education.

Comments made during the hearings were positive, and specific suggestions for improvement were made. Concerns expressed at all sites related to the degree of autonomy local districts would have in implementing the Essential Skills in Mathematics. Flagstaff participants were concerned that the revision was overly ambitious and ignored various problems which districts in the northern portion of the state faced. These problems included budget restrictions and high percentages of minority students. Flagstaff participants also expressed a concern that the state Board of Education, the Arizona Department of Education and the Arizona Legislature were burdening school districts with a revision when they had only recently managed to incorporate the 1984 list into their curriculum.

Final Revision

The Committee met on February 3, 1987, to consider all responses to the draft document. Committee members made recommendations that the draft be further revised to accommodate many of the responses; therefore, several changes were made to the draft the districts reviewed. These revisions included deletion of some skills from the list in response to concerns that some of them were too sophisticated for the grade level. The final revision, as submitted for approval by the state Board of Education on February 23, 1987, reflects the Committee's decisions regarding responses received from districts, schools and teachers across the state.

Introduction

The 1987 version of the *Arizona Essential Skills for Mathematics* has been interpreted by some as a checklist of isolated skills. The intent of the document was far more comprehensive.

Welcome to the 1992 edition of the *Arizona Essential Skills for Mathematics*. This document leaves intact the Arizona Essential Skills published in 1987. It cites current agreement among those within the mathematics education community as to how to prepare students for the 21st century.

The *Arizona Essential Skills for Mathematics* was first published in July 1987 by the Arizona Department of Education and is reflected in the mathematics component of the Arizona Student Assessment Program (ASAP). The delineation of these mathematics skills helped educators define what content and process skills students need. This document has subsequently been impacted by the 1989 publishing of the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics*. Their document, which has become known as the *NCTM Standards*, has affected the direction of mathematics education in this country.

Mathematics must be fully explored within the context of the real world. Although content is important, it is one's ability to problem solve that ultimately determines the outcomes of one's encounters with life. Problem solving, communication, reasoning and making mathematical connections — these aspects of mathematics thread through all grade levels. It is necessary to understand the interrelationships of these four processes. Without this understanding, it will not be possible to make the transition smoothly into the next century.

This new document is yours, mine, ours. Its format ties the Essential Skills to the *NCTM Standards* to help facilitate future curriculum development. From it you will learn what can be done — indeed, what changes must be willingly made — to prepare our young people to take charge of the world-to-be so that they can understand its problems and be prepared to work toward solutions. Where to begin? Probably not surprisingly, change begins within each of us.

Goals for Entering the 21st Century

As we enter the 21st century, we must make mathematical power a reality for all students.

Mathematical power is the capacity to do purposeful and worthwhile mathematical work. The critical manifestation of mathematical power lies in the student's ability to employ

- **Mathematical thinking** – *to use knowledge and understanding to analyze, conjecture, design, evaluate, formulate, generalize, investigate, model, predict, transform or verify*
- **Mathematical understanding** – *to use mathematical concepts and connections among concepts both within mathematics and across disciplines*
- **Tools and techniques** – *to efficiently and effectively solve mathematical problems; for example, using diagrams and tables, calculators and computers, manipulatives, and other concrete materials*
- **Communication Skills** – *to communicate results with various audiences and for various purposes.* (California Department of Education, 1991, 56)

To breathe life into mathematics education, three goals must be achieved:

1. Restructure mathematics curricula and teaching strategies to change mathematics from a static discipline to a dynamic process.
2. Set high standards of numeracy for **all** students.
3. Integrate student assessment with the learning process.

Rationale

It's Time to Change

"Other industrialized countries awakened 20 years ago to the significance of mathematics and science proficiency for their national well-being and began efforts to strengthen these components of education. We in the United States have been too slow to respond. As a result, we face harsh realities. ..."(Mathematical Science Education Board, 1991a, 3)

We have been dealing with minimal expectations and have attained mediocrity. We have used traditional mathematics curricula to achieve the results of the past, including our current levels of achievement. At a point in time where we are generating more questions than answers, are we preparing our students to ask the right questions in order to deal with constant change, or are we sending them into the world with the knowledge that there is only one right answer to every problem and the impression that there is only one way to find it? Are we helping our students to understand mathematics and its benefits effectively, so they can be shared by all?

We must build for a future of change. Some schools and their communities have already started. According to *Reshaping School Mathematics*, "The ruts of the old curriculum are being eroded by the waves of change sweeping across the landscape of mathematics education." (National Research Council 1990, 6) However, much more is needed if today's students are to become mathematically literate and use the language of mathematics comfortably and confidently in their daily lives.

Let's Change with the Times

Our mission for entering the 21st century is clear: ***Make mathematical power a reality for all students.*** Three goals have been identified for achievement. In order to gain a better understanding of these goals, a summary of each has been prepared. This information establishes where we are in mathematics education and where we must set our sights if our students are to attain mathematical power. The Committee cannot overemphasize the importance of each goal.

GOAL 1. *Restructure Mathematics Curricula and Teaching Strategies to Change Mathematics from a Static Discipline to a Dynamic Process*

Traditional curricula tends to break mathematical skills into minute pieces, presenting the skills as isolated fragments of knowledge with little carryover to the real world.

Teachers must teach students to think for themselves, as well as to work cooperatively with others. Curriculum and instruction must be changed to encourage students to explore, investigate, reason, conjecture, discuss, apply, create and communicate. "No single teaching method nor any single kind of learning experience can develop the varied mathematical abilities implied under the definition of mathematical power." (National Research Council, 1990b, 39)

GOAL 2. *Set High Standards of Numeracy for All Students*

Few U. S. students achieve levels of mathematics necessary to meet the demands of our society. Most students are relegated to mathematics classes that have minimal expectations and fail to equip them with skills necessary to prepare them for the world of work. Only in this country do people believe that learning mathematics is dependent on having special abilities. This myth, which has dominated mathematics education, can no longer be tolerated.

Teachers must promote a common core of mathematics for **all** students and a model of student-centered practice through group learning. Raising performance levels, keeping up with technological advances, and integrating mathematical concepts and critical thinking will help us make mathematical power a reality for all students.

"Everyone depends on the success of mathematics education; everyone is hurt when it fails." (National Research Council, 1989, 7)

GOAL 3. *Integrate Student Assessment with the Learning Process*

In changing mathematics from a static discipline to a dynamic process, it is evident that methods of assessment need to be redefined. They require developing ways to assess higher-order thinking processes. Setting high standards in numeracy for all necessitates focusing on what students know, as well as on what they need to know.

Assessment must be ongoing throughout the instructional process. Only then can a student's growth in understanding mathematical concepts and applying those concepts be fairly and accurately assessed. As students explore and develop their metacognitive skills (i.e., thinking about thinking), and use appropriate technology, they begin to communicate and illustrate their thinking processes using both oral and written language.

Making the Commitment

As we move to meet the goals set to make mathematical power a reality for all students, we must constantly monitor and adjust behavioral and curricular outcomes to meet the needs of students and society. It is not enough to strive to meet the present-day challenges without gauging our progress against our continually and rapidly changing society. As teachers and administrators, we are piloting a plane full of potential; and we must be willing to monitor and to make corrections to stay on course. Making adjustments does not say we have failed; rather it indicates that we have **learned** and are willing to use that knowledge to help our students grow — to help all of us reach our goal.

The technological needs of society and what we currently are teaching differ greatly. "Of all the influences that shape mathematics education, technology stands out as the one with greatest potential for revolutionary impact." (National Research Council, 1990b, 22) However, the use of technology in the mathematics classroom has been limited to reinforcing traditional curricula and has yet to approach its potential. As the age of technology grows, the gap is widening between what our students can do and the sophistication required to use the technology.

With the Information Age upon us, knowledge is exploding at a rate with which we can barely keep pace. Students must learn to use calculators, computers and other technological tools to solve problems and to go beyond where we presently are. Mathematics makes technology possible. "*Technology makes mathematics realistic. ...*" (National Research Council 1990b, 21)

"If we make a long-term commitment to the standards, ... if we approach the task with the will to persevere, if we are critical of the steps we take, ... we will make progress toward the goal of developing mathematical power for all students." (Mathematical Sciences Education Board, 1991a, 25) The benefits to the individual and to society are immeasurable.

Meeting the Challenge

In order to prepare students for the challenges of the future, we have explored changes that can be made in curriculum and methodology to bring the teaching of mathematics in line with what we now know. Here is a list of suggestions for change that can be realized at community, state and national levels:

- Make problem solving the focus of mathematics instruction and learning. Help students understand that they use mathematics to solve real-life problems. "All children learn and use mathematics better if it is derived from their reality, abstracted, practiced in enjoyable and effective ways, and applied to situations that are interesting and real to them." (Willoughby, 1990, 102)

- Emphasize that mathematics is a process, not a set of facts. Offer hands-on, experiential, risk-taking opportunities that build confidence and interest in that process. Show by example that there are no "right/wrong, quick answer" approaches to learning.

- Provide opportunities for using mathematical probability models and problem solving in **all** disciplines. Prompt students to practice and strengthen their communication skills as they make predictions, reason and solve problems.

- Use concrete materials that relate prior knowledge to new concepts. The 1987 Essential Skills document emphasizes that the use of concrete materials is critical to learning at all levels.

- Use appropriate technology in the classroom. "The *Standards* make clear that a textbook alone is not sufficient for teaching and learning mathematics. Classrooms must be equipped with calculators, computers, physical materials, and the associated 'software' to support these." (Trafton, 1989, 11-12)

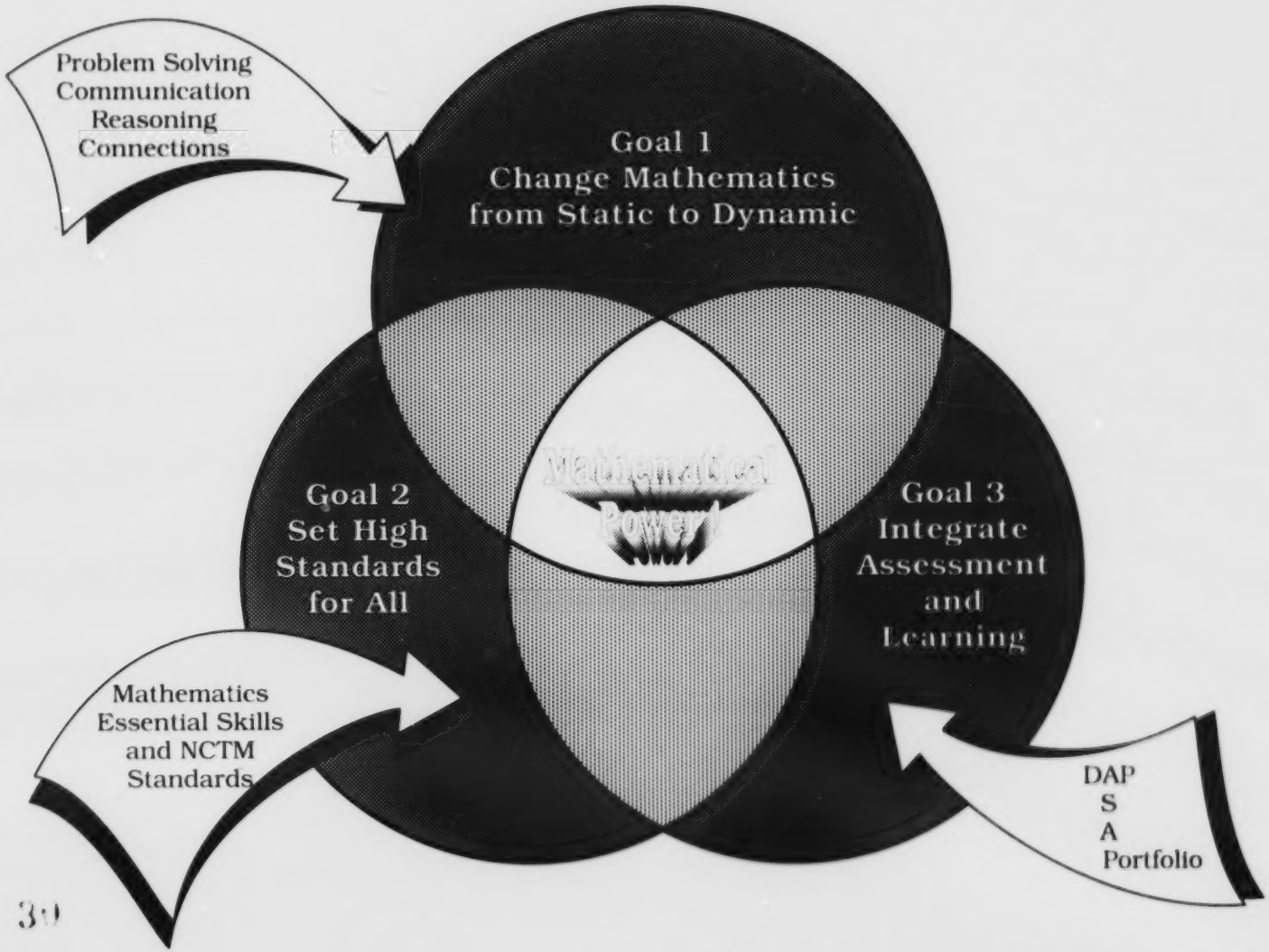
- Support the teacher as designer of new learning environments and provider of instruction, fostering student enjoyment and appreciation of mathematics.

- Select comprehensive standards for curricula and assessment. Use assessments not for comparing students but for helping students understand mathematics — for providing information with which to make instructional decisions.

- And perhaps the most critical point to be made: Show that restructuring is a worthwhile goal. Develop and reward partnerships of schools (including their boards), home and community with commitment to common goals and shared accountability for making change happen in education. Without a dedication to this partnership, change is impossible.

We've looked at the roles of the teacher and student, of mathematics education and learning, and of the community in the change process. What will be required for purposeful restructuring with built-in accountability? A critical variable will be perseverance. Its potential reward? Mathematical power!

Make Mathematical Power a Reality for ALL Students



From Goals to Action:

It's Time

In the past, the *Arizona Essential Skills for Mathematics* has been interpreted by some as a list of minimal expectations for student performance, which has generally been demonstrated by computing an algorithm out of context. In fact, the Essential Skills are multi-faceted. Therefore, the Committee aligned the Arizona Essential Skills to the *NCTM Standards* in order to show the many facets of the Essential Skills. By correlating them to the *Standards*, greater depth, meaning and understanding of the Arizona Essential Skills are illustrated.

The remainder of this document is divided into three sections. Each section addresses the three goals for preparing Arizona's students for the 21st century.

Goal 1: **Restructure mathematics curricula and teaching strategies to change mathematics from a static discipline to a dynamic process.**

Goal 2: **Set high standards of numeracy for all students.**

Goal 3: **Integrate student assessment with the learning process.**

All three goals have been correlated to the *NCTM Standards*. The relationship between each goal and the *Standards* and its impact on the Arizona Mathematics Essential Skills will be highlighted in each section.

The National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* is divided into two major sections. In the first section, 13-14 standards are identified for each grade level to describe what all students should be able to do in order to be mathematically empowered. The first four of these standards relate predominantly to the process of teaching mathematics, while the remainder relate more to the content and context of the concepts that should be taught. The second section of the document concentrates on program evaluation and student assessment.

GOAL 1. ***Restructure Mathematics Curricula and Teaching Strategies to Change Mathematics from a Static Discipline to a Dynamic Process***

The first four standards for all grade levels, K-12, are

- Mathematics as Problem Solving
- Mathematics as Communication
- Mathematics as Reasoning
- Mathematical Connections

These four standards permeate every Essential Skill in this document. They are the process, or vehicle, by which all the Essential Skills should be taught. Learning an Essential Skill out of the context of real-life application and without conceptual understanding is an exercise in futility. A comprehensive understanding of the Essential Skills cannot occur without integrating and applying these four standards throughout instruction.

"Problem solving should be the central focus of the mathematics curriculum." (*NCTM Standards*, 1989, 23) It is the vehicle that is necessary to create a full, rich classroom environment. It gives students the opportunity to communicate their thinking, develop higher-order reasoning skills and make connections.

Students need to have many opportunities to communicate — to discuss, describe and explain their thinking regarding mathematical concepts. This includes classroom and group discussions, diagrams or pictures representing mathematical thinking, and oral or written descriptions that clarify students' thinking about mathematical concepts.

Problem solving ... "places critical thinking at the heart of instruction." (*NCTM Standards*, 1989, 29) It invites students to reason — to think for themselves, apply different strategies and make the connections necessary to understand and apply the mathematical concepts.

Using a problem-solving approach to learning allows students to make connections. All students K-12 must be given the opportunity to explore new mathematical concepts using concrete materials. They should be assisted through the discovery process in order to help them develop within their own minds an understanding of the concepts. The teacher guides this process by helping the students clarify their thinking and make connections between the concrete and the abstract. By creating an understanding from within, students are more able to make connections between different mathematical strands and apply the concepts in real-life situations.

These standards bring mathematics to life and empower students mathematically. These processes create an environment that encourages our students to reason, communicate and make connections in problem-solving situations. By integrating these four standards into the instructional process, mathematics will be changed from a static discipline to a dynamic process.

When examining the remainder of the *NCTM Standards*, it becomes apparent that process cannot be separated from content; therefore, a more thorough explanation of each of the first four standards is included under Goal 2. Since the first four standards permeate all of the Essential Skills, the format on the matrix found in Goal 2 and the explanation of the first four standards vary slightly from the remainder of the standards.

GOAL 2. *Set High Standards of Numeracy for **All** Students*

All students can do mathematics, yet present-day mathematics curricula and instruction act as a sieve to weed out all but the very best. Most mathematics classes teach a traditional curriculum using traditional strategies — a practice that keeps most students from reaching their true potential.

Teachers need to re-examine what is being taught and how it is being taught. We all must begin to look at the mathematics curriculum differently and examine what is being done in the present-day curriculum that acts as a barrier to mathematics achievement.

As we look at what we can change to make that happen, we realize that we are talking not only about curriculum but also about instruction and perceptions toward mathematics.

In order to give every student an opportunity to achieve mathematical power, it is necessary for schools to

> "*Restructure the curriculum* so that long-term outcomes emphasize thinking, rather than memorizing facts.
> *Rethink how instruction is delivered* so that the structure of schools focuses on success, not failure, of students.
> *Help teachers improve their instructional strategies* so that they can succeed in teaching students with different learning styles, rates, and levels of motivation.
> *Create a school community that addresses the many needs of children and youth*, needs that affect how students learn and progress." (Lewis, 1989, 140)

How This Section Is Organized

As in the 1987 *Arizona Essential Skills for Mathematics*, this section of the document is divided into three parts: K-3, 4-8 and 9-12. In each part, the Essential Skills are presented in their original order and are correlated, in a matrix format, to the *Standards* set forth by the National Council of Teachers of Mathematics.

The first four standards permeate all of the Essential Skills, and the boxes have been shaded to illustrate their relationship. Direct relationships between content skills are indicated by using an X.

Both the Arizona Essential Skills and the *NCTM Standards* are open to a wide range of interpretation. If no relationship has been delineated, this does not eliminate the possibility of correlation. Further, it is not the Committee's intent to restrict the transfer from any one area to another. (In fact, there were members of the Committee who felt they could not limit the possibility of transfer and that virtually all areas should be marked.)

To help the user become familiar with the **multifaceted** relationship of the *NCTM Standards* to the Arizona Essential Skills, we have illustrated a portion of the K-3 matrix (see page 11). As you can see on the matrix, skill I-3, *Demonstrate an understanding of the meaning of the four basic operations*, can no longer be taught as an isolated algorithm. Rather, students must be given the opportunity to apply the skill in problem-solving situations, communicate their understanding, develop logical reasoning skills, and make mathematical connections through real-world applications. Applying this skill in the context of estimation, concepts of whole number operations, and whole number computation enables the student to demonstrate a more complete understanding of the skill. Further, students should be able to apply these operations throughout the remaining standards.

Arizona Mathematics Essential Skills

Mathematical Outcomes Expected of Primary Students

Mathematics instruction in Grades K-3 should ensure that students completing Grade 3 have experiences which enable them to perform the following outcomes:

I. Number

1. Count by ones, two, fives and tens.
2. Use cardinal and ordinal numbers to compare and order quantities.
3. Demonstrate an understanding of the meaning of the four basic operations.
4. Have facility with basic addition, subtraction and multiplication facts.
5. Use concrete materials or models to demonstrate an understanding of place value.

Standards														
1 Problem Solving	2 Communication	3 Reasoning	4 Connections	5 Estimation	6 Number Sense	7 Concepts of Whole Number Operation	8 Whole Number Computation	9 Geometry and Spatial Sense	10 Measurement	11 Statistics and Probability	12 Fractions and Decimals	13 Patterns and Relationships	
X	X	X	X		X							X	1
X	X	X	X		X								2
X	X	X	X	X		X	X						3
X	X	X	X	X		X	X						4
X	X	X	X		X						X		5

Following the matrices are charts indicating student outcomes and examples of performance indicators for the remaining standards. The student outcomes listed with each standard are the Essential Skills that have been directly correlated to that standard. The examples of indicators refer specifically to how students might demonstrate an understanding of the Essential Skills in light of that standard only.

Standard 5: Estimation

Estimation is an essential component of mathematics since mathematics involves more than exactness. Estimation interacts with number sense and spatial sense to assist students in developing insights into concepts and procedures, enabling them to work flexibly with number and measurement and to evaluate the reasonableness of results.

Student Outcomes	Examples of Indicators
Demonstrate an understanding of the meaning of the four basic operations. (I-3)	◆ Apply a variety of estimation strategies.
Have facility with basic addition, subtraction and multiplication facts. (I-4)	◆ Recognize when estimation is appropriate.
Estimate answers to computational problems. (I-8)	◆ Determine the reasonableness of results.
Add or subtract two three-digit whole numbers. (I-9)	◆ Apply estimation in working with quantities, measurement, computation and problem-solving.
Recognize and count money. (I-16)	

It should be noted that the Arizona Essential Skills are presented in a K-3 and 4-8 format, while the *NCTM Standards* have a K-4 and 5-8 format. Since fourth grade is a year of transition from primary to intermediate level, the Committee believes that the non-parallel focus between the Essential Skills and the *NCTM Standards* is to our benefit. The *NCTM Standards* addresses the primary focus for fourth grade, while the Essential Skills are tied directly to the 5-8 section in this document, setting the foundation and direction for the transition from primary to intermediate levels of mathematics. It is imperative that fourth grade curriculum and instructional practices interweave the fundamentals of both the K-3 and 4-8 sections that follow.

K-3
MATHEMATICS
ESSENTIAL SKILLS

BEST COPY AVAILABLE

Arizona Mathematics Essential Skills

Mathematical Outcomes Expected of Primary Students

Mathematics instruction in Grades K-3 should ensure that students completing Grade 3 have experiences which enable them to perform the following outcomes:

I. Number

	Standards													
	1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Estimation	6. Number Sense	7. Concepts of Whole Number Operation	8. Whole Number Computation	9. Geometry and Spatial Sense	10. Measurement	11. Statistics and Probability	12. Fractions and Decimals	13. Patterns and Relationships	
1. Count by ones, two, fives and tens.	X	X	X	X		X							X	1
2. Use cardinal and ordinal numbers to compare and order quantities.	X	X	X	X		X								2
3. Demonstrate an understanding of the meaning of the four basic operations.	X	X	X	X	X		X	X						3
4. Have facility with basic addition, subtraction and multiplication facts.	X	X	X	X	X		X	X						4
5. Use concrete materials or models to demonstrate an understanding of place value.	X	X	X	X		X						X		5
6. Explore the concepts of multiplication and division with concrete materials.	X	X	X	X			X							6
7. Read and write the number represented when objects are grouped by hundreds, tens and ones.	X	X	X	X		X								7
8. Estimate answers to computational problems.	X	X	X	X	X	X	X	X						8
9. Add or subtract two three-digit whole numbers.	X	X	X	X	X			X						9
10. Choose the appropriate operation to be used in a given situation.	X	X	X	X			X	X						10
11. Interpret word problems by using role playing, pictures and models.	X	X	X	X			X							11
12. Write mathematical sentences to represent a situation.	X	X	X	X			X	X				X		12

Arizona Mathematics Essential Skills (continued)

	Standards													
	1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Estimation	6. Number Sense	7. Concepts of Whole Number Operation	8. Whole Number Computation	9. Geometry and Spatial Sense	10. Measurement	11. Statistics and Probability	12. Fractions and Decimals	13. Patterns and Relationships	
I. Number (continued)														
13. Use informally the properties of commutativity, associativity and identity.	X	X	X	X			X							13
14. Use concrete materials to recognize, represent and compare halves, thirds and fourths.	X	X	X	X								X		14
15. Recognize fractional equivalents for halves, fourths and tenths.	X	X	X	X								X		15
16. Recognize and count money.	X	X	X	X	X							X		16
17. Use money to represent and compare decimal values.	X	X	X	X								X		17
II. Measurement														
1. Use nonstandard, metric and English units of measure to estimate and measure length, volume and weight.	X	X	X	X		X	X		X	X		X		1
2. Use digital and conventional clocks to tell time.	X	X	X	X	X					X		X		2
3. Read and interpret Celsius and Fahrenheit temperatures on thermometers.	X	X	X	X	X	X				X				3
4. Choose an appropriate unit of measure in a given situation.	X	X	X	X		X				X				4
5. Use a variety of measurement instruments.	X	X	X	X						X				5

Arizona Mathematics Essential Skills (continued)

	Standards														
	1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Estimation	6. Number Sense	7. Concepts of Whole Number Operation	8. Whole Number Computation	9. Geometry and Spatial Sense	10. Measurement	11. Statistics and Probability	12. Fractions and Decimals	13. Patterns and Relationships	
III. Geometry														
1. Use terminology appropriate to the grade level.	X	X	X	X					X					1
2. Use visual attributes, concrete materials and appropriate vocabulary to identify, classify and describe common geometric figures and models.	X	X	X	X					X					2
3. Use several geometric shapes to make other geometric shapes.	X	X	X	X					X					3
4. Decide whether figures are congruent and whether they are similar.	X	X	X	X					X					4
5. Explore the filling of space using manipulatives.	X	X	X	X					X					5
6. Use manipulative materials to develop the concepts of point, line and line segment.	X	X	X	X					X					6
IV. Patterns and Relations														
1. Identify, describe and extend a pattern in a sequence of objects.	X	X	X	X									X	1
2. Use a concrete model to create a pattern and represent that pattern symbolically.	X	X	X	X									X	2
3. Describe the relationship given in a table of numbers derived from a sequence of objects.	X	X	X	X									X	3
4. Determine a location by using ordered pairs of numbers on a rectangular grid.	X	X	X	X									X	4

Arizona Mathematics Essential Skills (continued)

	Standards													
	1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Estimation	6. Number Sense	7. Concepts of Whole Number Operation	8. Whole Number Computation	9. Geometry and Spatial Sense	10. Measurement	11. Statistics and Probability	12. Fractions and Decimals	13. Patterns and Relationships	
V. Data Analysis and Probability														
1. Collect, organize, represent and interpret data derived from surveys and experiments conducted by the students.	X	X	X	X							X			1
2. Create and interpret concrete, pictorial and symbolic graphs.	X	X	X	X							X			2
3. Perform simple activities involving probability.	X	X	X	X	X						X		X	3
VI. Analytical Reasoning														
1. Classify and sort objects by observing relationships and making generalizations.	X	X	X	X					X					1
2. Make reasonable or logical conjectures and conclusions about situations with concrete materials, using such words as these: *and, or, if ... then, all, some, none, not* and *out of.*	X	X	X	X	X				X					2

Standard 1: Mathematics as Problem Solving

Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned.

Student Outcomes

Use all of the mathematics Essential Skills in a problem-solving context.

50

Examples of Indicators

- Use problem-solving approaches to investigate and understand mathematical concept.
- Formulate problems from everyday and mathematical situations.
- Develop and apply strategies to solve a wide variety of problems.
- Verify and interpret results with respect to the original problem.
- Demonstrate confidence in using mathematics meaningfully.

51

Standard 2: Mathematics as Communication

Communicating mathematical thinking provides a vehicle for students to explore their ideas about mathematical concepts, clarify and strengthen their understanding, and construct links between concepts and real-life situations. Students need to be given ample time to explore, investigate, describe and discuss their thinking as they work with mathematical concepts.

Student Outcomes

Communicate and explain thinking about all of the mathematics Essential Skills in a variety of contexts.

52

Examples of Indicators

- Relate physical materials, pictures and diagrams to mathematical ideas.
- Reflect on and clarify their thinking about mathematical ideas and situations.
- Relate their everyday language to mathematical language and symbols.
- Demonstrate how representing, discussing, reading, writing and listening to mathematics are a vital part of learning and using mathematics.

53

Standard 3: Mathematics as Reasoning

Mathematics is reasoning. One cannot do mathematics without reasoning. Critical thinking must be placed at the heart of instruction by providing students with many opportunities to explore and evaluate problems, make conjectures and explain and justify their thinking, and solve problems in a variety of ways. It is as important for students to explain how they solved a problem as the solution itself.

Student Outcomes

Use exploration, discussion and conjecture to evaluate and justify thinking about each of the mathematics Essential Skills within the context of a problem or in real-life situations.

Examples of Indicators

- Draw logical conclusions about mathematics.
- Use models, known facts, properties and relationships to explain their thinking.
- Justify their answers and solution processes.
- Use patterns and relationships to analyze mathematical situations.

Standard 4: Mathematical Connections

Mathematical connections are the threads that hold the fabric of mathematics together. Students need to make mathematical connections between the concrete and the abstract, between the different mathematical strands, and between the classroom problems and the real world.

Student Outcomes

Make connections between the mathematics Essential Skills and their many levels of application.

Examples of Indicators

- Link conceptual and procedural knowledge.
- Relate various representations of concepts or procedures to one another.
- Recognize relationships among different topics in mathematics.
- Use mathematics in other curriculum areas.
- Use mathematics in real-life activities.

Standard 5: Estimation

Estimation is an essential component of mathematics since mathematics involves more than exactness. Estimation interacts with number sense and spatial sense to assist students in developing insights into concepts and procedures, enabling them to work flexibly with number and measurement and to evaluate the reasonableness of results.

Student Outcomes

Demonstrate an understanding of the meaning of the four basic operations. (I-3)

Have facility with basic addition, subtraction and multiplication facts. (I-4)

Estimate answers to computational problems. (I-8)

Add or subtract two three-digit whole numbers. (I-9)

Recognize and count money. (I-16)

Use nonstandard, metric and English units of measure to estimate and measure length, volume and weight. (II-1)

Use digital and conventional clocks to tell time. (II-2)

Read and interpret Celsius and Fahrenheit temperatures on thermometers. (II-3)

Perform simple activities involving probability. (V-3)

Make reasonable or logical conjectures and conclusions about situations with concrete materials, using such words as these: *and, or, if ... then, all, some, none, not* and *out of*. (VI-2)

Examples of Indicators

- Apply a variety of estimation strategies.
- Recognize when estimation is appropriate.
- Determine the reasonableness of results.
- Apply estimation in working with quantities, measurement, computation and problem solving.

Standard 6: Number Sense and Numeration

Number sense and numeration are the development of whole number concepts and skills through the use of concrete material and physical manipulations. This enables children to build and extend number relationships and helps them to develop a link between their world and the world of mathematics.

Student Outcomes

Count by ones, twos, fives and tens. (I-1)

Use cardinal and ordinal numbers to compare and order quantities. (I-2)

Use concrete materials or models to demonstrate an understanding of place value. (I-5)

Read and write the number represented when objects are grouped by hundreds, tens and ones. (I-7)

Estimate answers to computational problems. (I-8)

Use nonstandard, metric and English units of measure to estimate and measure length, volume and weight. (II-1)

Read and interpret Celsius and Fahrenheit temperatures on thermometers. (II-3)

Choose an appropriate unit of measure in a given situation. (II-4)

60

Examples of Indicators

- Use concrete materials to construct and explain number meanings and operations.
- Interpret the multiple uses of numbers encountered in the real world.
- Relate counting, grouping and place value concepts within the base 10 numeration system.

61

Standard 7: Concepts of Whole Number Operations

Understanding the fundamental operations of addition, subtraction, multiplication and division is central to knowing mathematics. It is important for children to build an awareness of models and properties of these operations, seeing the relationships among them.

Student Outcomes

Demonstrate an understanding of the meaning of the four basic operations. (I-3)

Have facility with basic addition, subtraction and multiplication facts. (I-4)

Explore the concepts of multiplication and division with concrete materials. (I-6)

Estimate answers to computational problems. (I-8)

Choose the appropriate operation to be used in a given situation. (I-10)

Interpret word problems by using role playing, pictures and models. (I-11)

Write mathematical sentences to represent a situation. (I-12)

Use informally the properties of commutativity, associativity and identity. (I-13)

62

Examples of Indicators

- Model, using concrete materials, the meaning for the operations.
- Relate the mathematical language and symbolism of operations to problem situations and informal language.
- Demonstrate operation sense by role playing or modeling a wide variety of problem situations.

63

Standard 8: Whole Number Computation

Whole number computation encompasses a variety of ways for children to solve problems, from developmental levels using concrete materials to solving problems by using abstract algorithms. This strengthens a child's reasoning skills, mathematical insight and confidence to do mathematics.

Student Outcomes

Demonstrate an understanding of the meaning of the four basic operations. (I-3)

Have facility with basic addition, subtraction and multiplication facts. (I-4)

Estimate answers to computational problems. (I-8)

Add or subtract two three-digit whole numbers. (I-9)

Choose the appropriate operation to be used in a given situation. (I-10)

Write mathematical sentences to represent a situation. (I-12)

64

Examples of Indicators

- Use concrete materials to demonstrate understanding of mathematical procedures.
- Demonstrate reasonable proficiency with basic facts and algorithms.
- Use a variety of mental computation and estimation techniques.
- Use calculators in appropriate computational situations.
- Select and use computation techniques appropriate to specific problems, and determine whether the results are reasonable.

65

Standard 9: Geometry and Spatial Sense

Geometry and spatial sense help a student to represent and describe in an orderly manner our inherently geometric world. Understanding should grow naturally from exploration and experience with two- and three-dimensional figures in problem-solving situations.

Student Outcomes

Use nonstandard, metric and English units of measure to estimate and measure length, volume and weight. (II-1)

Use terminology appropriate to the grade level. (III-1)

Use visual attributes, concrete materials and appropriate vocabulary to identify, classify and describe common geometric figures and models. (III-2)

Use several geometric shapes to make other geometric shapes. (III-3)

Decide whether figures are congruent and whether they are similar. (III-4)

Explore the filling of space using manipulatives. (III-5)

Use manipulative materials to develop the concepts of point, line and line segment. (III-6)

Classify and sort objects by observing relationships and making generalizations. (VI-1)

Make reasonable or logical conjectures and conclusions about situations with concrete materials, using such words as these: *and, or, if ... then, all, some, none, not* and *out of*. (VI-2)

66

Examples of Indicators

- Describe, model, draw and classify shapes.
- Investigate and predict the results of combining, subdividing and changing shapes.
- Relate geometric ideas to number and measurement.
- Recognize geometry in the real world.

67

Standard 10: Measurement

Measurement is comparison. A unit appropriate for the attribute to be measured is selected, the comparison made, and the results reported. Students must develop an informal understanding of the process of measurement with nonstandard units before standard instruments are introduced.

Student Outcomes

Use nonstandard, metric and English units of measure to estimate and measure length, volume and weight. (II-1)

Use digital and conventional clocks to tell time. (II-2)

Read and interpret Celsius and Fahrenheit temperatures on thermometers. (II-3)

Choose an appropriate unit of measure in a given situation. (II-4)

Use a variety of measurement instruments. (II-5)

Examples of Indicators

- Use a variety of measurement instruments to determine length, capacity, weight, area, volume, time, temperature and angle.
- Develop an understanding of the process of measuring.
- Use formal and informal measurements in problem solving and everyday situations.

Standard 11: Statistics and Probability

Statistics and probability are tools for problem solving. By collecting, organizing, displaying and thinking about data in many ways, we learn to use numbers to describe and interpret the world around us.

Student Outcomes

Collect, organize, represent and interpret data derived from surveys and experiments conducted by the students. (V-1)

Create and interpret concrete, pictorial and symbolic graphs. (V-2)

Perform simple activities involving probability. (V-3)

Examples of Indicators

- Collect, organize and describe data.
- Construct, read and interpret displays of data.
- Formulate and solve problems that involve collecting and analyzing data.
- Explore concepts of chance and make appropriate conjectures.

Standard 12: Fractions and Decimals

Fractions and decimals are necessary to describe the real world. Students need opportunities to explore fractional relationships and build an understanding of order and equivalence. Students build understanding through the use of concrete materials, diagrams and real-world situations.

Student Outcomes

Use concrete materials or models to demonstrate an understanding of place value. (I-5)

Write mathematical sentences to represent a situation. (I-12)

Use concrete materials to recognize, represent and compare halves, thirds and fourths. (I-14)

Recognize fractional equivalents for halves, fourths and tenths. (I-15)

Recognize and count money. (I-16)

Use money to represent and compare decimal values. (I-17)

Use nonstandard, metric and English units of measure to estimate and measure length, volume and weight. (II-1)

Use digital and conventional clocks to tell time. (II-2)

Examples of Indicators

- Use concepts of fractions, mixed numbers and decimals to solve problem situations.
- Use number sense in order to determine reasonableness of answers.
- Use models to explain the relationship between fractions and decimals and to find equivalent fractions.
- Use models to explain operations for fractions and decimals.
- Apply fractions and decimals in real-life situations.

Standard 13: Patterns and Relationships

Patterns and relationships enable students to classify and organize information mathematically. Students should be given opportunities to focus on the regularities of events, shapes, designs and sets of numbers in order to identify patterns.

Student Outcomes

Count by ones, twos, fives and tens. (I-1)

Identify, describe and extend a pattern in a sequence of objects. (IV-1)

Use a concrete model to create a pattern and represent that pattern symbolically. (IV-2)

Describe the relationship given in a table of numbers derived from a sequence of objects. (IV-3)

Determine a location by using ordered pairs of numbers on a rectangular grid. (IV-4)

Perform simple activities involving probability. (V-3)

Examples of Indicators

- Recognize, describe, extend and create a wide variety of patterns using concrete and pictorial representations.
- Represent and describe mathematical relationships.
- Use variables and open sentences to express relationships.
- Classify and organize information based on patterns identified.

Standard 13: Patterns and Relationships

Patterns and relationships enable students to classify and organize information mathematically. Students should be given opportunities to focus on the regularities of events, shapes, designs and sets of numbers in order to identify patterns.

Student Outcomes

Count by ones, twos, fives and tens. (I-1)

Identify, describe and extend a pattern in a sequence of objects. (IV-1)

Use a concrete model to create a pattern and represent that pattern symbolically. (IV-2)

Describe the relationship given in a table of numbers derived from a sequence of objects. (IV-3)

Determine a location by using ordered pairs of numbers on a rectangular grid. (IV-4)

Perform simple activities involving probability. (V-3)

Examples of Indicators

- Recognize, describe, extend and create a wide variety of patterns using concrete and pictorial representations.
- Represent and describe mathematical relationships.
- Use variables and open sentences to express relationships.
- Classify and organize information based on patterns identified.

4-8
MATHEMATICS
ESSENTIAL SKILLS

BEST COPY AVAILABLE

Arizona Mathematics Essential Skills

Mathematical Outcomes Expected of Middle School Students

Mathematics instruction in Grades 4-8 should ensure that students completing Grade 8 have experiences which enable them to perform the following outcomes:

I. Number

		Standards													
		1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Number & Number Rel.	6. Number Syst. & Number Theory	7. Computations and Estimations	8. Patterns and Functions	9. Algebra	10. Statistics	11. Probability	12. Geometry	13. Measurement	
1.	Use concrete materials and illustrative models to represent place value in whole and decimal numbers, fractions and their equivalents and percents.	X	X	X	X	X	X								1
2.	Read and write whole numbers, integers, common fractions and decimal fractions.	X	X	X	X	X									2
3.	Use appropriate symbols and order relations when comparing whole numbers, integers, common fractions and decimal fractions.	X	X	X	X	X	X								3
4.	Identify and use number properties to simplify expressions and calculations, *i.e.*, commutative, associative, distributive, identity of one and zero, rule of order of operations.	X	X	X	X		X	X							4
5.	Convert numbers in base 10 notation to and from scientific notation.	X	X	X	X	X	X	X							5
6.	Convert numbers in base 10 notation to and from expanded notation.	X	X	X	X	X	X	X							6
7.	Understand the square root of a number and the methods of finding it or its approximation.	X	X	X	X	X	X	X							7
8.	List the prime factorization of a whole number and ultimately write it as a numerical expression involving positive exponents.	X	X	X	X	X	X	X							8
9.	Understand greatest common factor and least common multiple and their applications.	X	X	X	X	X	X	X							9
10.	Find equivalent fractions.	X	X	X	X	X	X	X							10

80

81

Arizona Mathematics Essential Skills (continued)

	Standards													
	1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Number & Number Rel.	6. Number Syst. & Number Theory	7. Computations and Estimations	8. Patterns and Functions	9. Algebra	10. Statistics	11. Probability	12. Geometry	13. Measurement	
1. Number (continued)														
11. Recall addition, subtraction, multiplication and division facts with reasonable speed.	X	X	X	X			X							11
12. Use basic operations with whole numbers, integers, common fractions and decimal fractions.	X	X	X	X		X	X							12
13. Use various techniques of mental arithmetic.	X	X	X	X		X	X							13
14. Estimate to predict answers to computational problems and to check for reasonableness of answers.	X	X	X	X		X	X							14
15. Use a calculator in appropriate situations.	X	X	X	X			X							15
16. Use the additive and multiplicative laws of exponents.	X	X	X	X	X	X	X							16
17. Use ratios and proportions to solve problems.	X	X	X	X	X	X	X							17
18. Convert among percent, fractional and decimal equivalents.	X	X	X	X	X	X	X							18
19. Solve for the unknown in a percent problem.	X	X	X	X		X	X							19
20. Interpret problems, translate to numerical expressions, choose an appropriate method of calculation and solve.	X	X	X	X		X	X							20
21. Solve problems involving money.	X	X	X	X		X	X							21
22. Round off numbers to a specified place value.	X	X	X	X	X	X	X							22

Arizona Mathematics Essential Skills (continued)

II. Measurement

	Standards													
	1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Number & Number Rel	6. Number Syst. & Number Theory	7. Computations and Estimations	8. Patterns and Functions	9. Algebra	10. Statistics	11. Probability	12. Geometry	13. Measurement	
1. Know that all measurement is approximate.	X	X	X	X									X	1
2. Select the appropriate measuring instrument.	X	X	X	X									X	2
3. Add and subtract measures of time.	X	X	X	X			X						X	3
4. Compare temperatures and determine the amount of change.	X	X	X	X			X						X	4
5. Measure length, volume and weight in both English and metric units.	X	X	X	X									X	5
6. Convert English and metric measurement units to equivalent units within the given system.	X	X	X	X			X						X	6
7. Be aware of approximate equivalent measurements between English and metric units.	X	X	X	X									X	7
8. Use an appropriate formula to calculate the perimeter and area of polygons and the circumference of a circle.	X	X	X	X			X		X			X	X	8
9. Use an appropriate formula to calculate the volume and surface area of the following solids: spheres, prisms, pyramids, cylinders and cones.	X	X	X	X			X		X			X	X	9
10. Use a protractor to measure and draw angles.	X	X	X	X								X	X	10
11. Use an appropriate formula to determine a measurement when a direct measurement tool is unavailable.	X	X	X	X			X		X			X	X	11
12. Estimate lengths, areas, volumes and weights, and check by measuring in English and metric units.	X	X	X	X								X	X	12
13. Solve problems with measurements including distance, weight, time, area, capacity and temperature.	X	X	X	X			X					X	X	13

Arizona Mathematics Essential Skills (continued)

	Standards													
	1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Number & Number Rel.	6. Number Syst. & Number Theory	7. Computations and Estimations	8. Patterns and Functions	9. Algebra	10. Statistics	11. Probability	12. Geometry	13. Measurement	
III. Geometry														
1. Use terminology appropriate to the grade level.	X	X	X	X								X		1
2. Recognize two- and three-dimensional figures by their shapes and identify their component parts and associated properties.	X	X	X	X				X				X		2
3. Through observation, measurement, drawing and modeling, identify geometric properties such as symmetry, congruence, similarity, parallelism and perpendicularity.	X	X	X	X				X				X	X	3
4. Visualize, draw and construct two- and three-dimensional figures through concrete experiences.	X	X	X	X								X	X	4
5. Utilize geometric principles and ideas to solve problems.	X	X	X	X			X		X			X	X	5
6. Experiment with, discover and apply the known formulas of geometry.	X	X	X	X			X	X	X			X	X	6
7. Understand pi and its importance in the geometry of the circle.	X	X	X	X			X	X	X			X	X	7
8. Recognize and classify angles as well as identify relationships between angles.	X	X	X	X				X				X	X	8
9. Draw and/or construct a variety of shapes having the same area.	X	X	X	X								X	X	9
IV. Patterns and Relations														
1. Organize data to show relationships in tables and graphs.	X	X	X	X	X			X	X	X				87

Arizona Mathematics Essential Skills (continued)

	Standards													
	1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Number & Number Rel.	6. Number Syst. & Number Theory	7. Computations and Estimations	8. Patterns and Functions	9. Algebra	10. Statistics	11. Probability	12. Geometry	13. Measurement	
IV. Patterns and Relations (continued)														
2. Interpret data from tables and charts to determine relationships.	X	X	X	X	X			X	X	X	X			2
3. Determine an expression or an equation for a relationship and then evaluate or solve it for a given value of the variable.	X	X	X	X				X	X	X				3
4. Recognize or find a specific pattern occurring in a sequence of numbers.	X	X	X	X	X			X	X	X				4
5. Extend patterns and create new ones.	X	X	X	X	X	X	X	X	X					5
6. Graph ordered pairs to show a pattern or relationship.	X	X	X	X	X			X	X					6
7. Understand the relationship between angles that are complementary, supplementary and vertical.	X	X	X	X								X		7
8. Identify and understand the relationships of angles formed by a transversal of two parallel lines.	X	X	X	X								X		8
9. Understand the relationships occurring between similar figures.	X	X	X	X				X				X		9
V. Data Analysis and Probability														
1. Collect and organize data using lists, tables and graphs.	X	X	X	X				X	X	X				1
2. Interpret, analyze and draw conclusions from graphs.	X	X	X	X				X	X	X	X			2
3. Take polls and surveys.	X	X	X	X						X				3

Arizona Mathematics Essential Skills (continued)

	Standards													
	1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Number & Number Rel.	6. Number Syst. & Number Theory	7. Computations and Estimations	8. Patterns and Functions	9. Algebra	10. Statistics	11. Probability	12. Geometry	13. Measurement	
V. Data Analysis and Probability (continued)														
4. Predict and record the probability of events from simple experiments.	X	X	X	X				X		X	X			4
5. Express probability as a ratio or fraction.	X	X	X	X			X				X			5
6. Find the mean, median, mode and range for a set of data.	X	X	X	X			X			X				6
7. Determine the probability of simple events and draw conclusions or make interpretations.	X	X	X	X				X		X	X			7
8. Find the empirical probability of an event from a sample of observed outcomes.	X	X	X	X				X		X	X			8
9. Find the probability of complementary events and of mutually exclusive events.	X	X	X	X				X		X	X			9
10. Generate a frequency distribution for a given list of data.	X	X	X	X						X	X			10
11. Use a list or tree diagram to count permutations or combinations.	X	X	X	X						X	X			11
VI. Analytical Reasoning														
1. Recognize and identify numerical and geometric patterns and sequences.	X	X	X	X	X			X	X			X		1
2. Classify and categorize sets of numbers or groups of geometric shapes.	X	X	X	X	X			X				X		2
3. Using a given attribute, find similarities and differences among geometric shapes or designs.	X	X	X	X								X		3

Arizona Mathematics Essential Skills (continued)

	Standards													
	1. Problem Solving	2. Communication	3. Reasoning	4. Connections	5. Number & Number Rel.	6. Number Syst. & Number Theory	7. Computations and Estimations	8. Patterns and Functions	9. Algebra	10. Statistics	11. Probability	12. Geometry	13. Measurement	
VI. Analytical Reasoning (continued)														
4. Appropriately use terms such as *and, or, not, only* and *if … then*, in a mathematical sense.	X	X	X	X	X	X	X	X	X	X	X	X		4
5. Provide specific examples of a given numerical or geometric principle.	X	X	X	X		X	X		X			X		5
6. Provide a counterexample of a condition that is not always true.	X	X	X	X							X	X		6
7. Perform tasks involving inductive and deductive reasoning.	X	X	X	X	X	X	X	X	X	X	X	X		7
VII. Algebra														
1. Understand the meaning of "variable" and "constant."	X	X	X	X					X					1
2. Be familiar and proficient with the standard order of operations.	X	X	X	X		X	X		X					2
3. Represent mathematical relationships using variables.	X	X	X	X				X	X					3
4. Simplify polynomial expressions.	X	X	X	X					X					4
5. Evaluate simple algebraic expressions by substituting values for the variables.	X	X	X	X			X		X					5
6. Solve two-step equations and inequalities using integers, fractions and decimals.	X	X	X	X			X		X					6

Standard 1: Mathematics as Problem Solving

Problem solving is the process by which students experience the power of mathematics and its usefulness in the world around them. Through different problem-solving activities, students can use a variety of strategies and mathematical tools to solve real-world problems. Children need to be given many opportunities to work with other students in analyzing, organizing and solving problems, using "messy data" from the real world.

Student Outcomes

Use a variety of problem-solving strategies to solve real-world problems involving the mathematics Essential Skills.

94

Examples of Indicators

- Use problem-solving approaches to investigate and understand mathematical content.
- Formulate problems from situations within and outside mathematics.
- Develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems.
- Verify and interpret results with respect to the original problem situation.
- Generalize solutions and strategies to new problem situations.

95

Standard 2: Mathematics as Communication

In grades 4-8, communication becomes even more important. Students need to discuss, explain, conjecture, justify and validate their thinking in order to deepen their understanding of mathematical concepts. Students need to use that understanding to analyze and relate mathematics to the real world.

Student Outcomes

Communicate mathematically their understanding of the mathematics Essential Skills and relate that understanding to real-world applications.

Examples of Indicators

- Model situations using oral, written, concrete, pictorial, graphical and algebraic methods.
- Reflect on and clarify thinking about mathematical ideas and situations.
- Develop common understanding of mathematical ideas, including the role of definitions.
- Use the skills of reading, listening and viewing to interpret and evaluate mathematical ideas.
- Discuss mathematical ideas and make conjectures and convincing arguments.

Standard 3: Mathematics as Reasoning

Reasoning is fundamental to knowing and doing mathematics. Students need to be given ample time to explore and analyze mathematical concepts and problem-solving situations, and to make conjectures and construct valid arguments pertaining to those problems.

Student Outcomes

Use mathematical reasoning to gain an understanding of the mathematics Essential Skills and use this understanding to solve a variety of problems.

Examples of Indicators

- Recognize and apply deductive and inductive reasoning.
- Understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs.
- Make and evaluate mathematical conjectures and arguments.
- Validate thinking.
- Use persuasion and the power of reasoning as a part of mathematics.

Standard 4: Mathematical Connections

Mathematical connections focuses on mathematics as an integrated whole as opposed to a list of isolated facts that need to be memorized. Students need to be given ample opportunities to explore the interconnections between mathematical concepts, as well their relationships to the real world. If students are to be empowered mathematically, they must be able to apply mathematics using a variety of strategies in different problem situations.

Student Outcomes

Make mathematical connections between mathematics Essential Skills and problem situations and demonstrate their understanding that mathematics is an integrated whole.

Examples of Indicators

- Explore problems and describe results using graphical, numerical, physical, algebraic and verbal mathematical models or representations.
- Use a mathematical idea to further understanding of other mathematical ideas.
- Apply mathematical thinking and modeling to solve problems that arise in other disciplines.
- Apply mathematical thinking and modeling to solve problems reflective of our culture and society.

Standard 5: Number and Number Relationships

Number and number relationships are the understanding of multiple representations of whole numbers, fractions, decimals, integers and rational numbers through concrete experiences in the context of real-world situations.

Student Outcomes

Use concrete materials and illustrative models to represent place value in whole and decimal numbers, fractions and their equivalents and percents. (I-1)

Read and write whole numbers, integers, common fractions and decimal fractions. (I-2)

Use appropriate symbols and order relations when comparing whole numbers, integers, common fractions and decimal fractions. (I-3)

Convert numbers in base 10 notation to and from scientific notation. (I-5)

Convert numbers in base 10 notation to and from expanded notation. (I-6)

Understand the square root of a number and the methods of finding it or its approximation. (I-7)

List the prime factorization of a whole number and ultimately write it as a numerical expression involving positive exponents. (I-8)

Understand greatest common factor and least common multiple and their applications. (I-9)

Find equivalent fractions. (I-10)

Use the additive and multiplicative laws of exponents. (I-16)

Use ratios and proportions to solve problems. (I-17)

102

Examples of Indicators

- Represent and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential and scientific notation) in real-world and mathematical problem situations.
- Apply ratios, proportions and percents in a wide variety of situations.
- Design, investigate and demonstrate relationships among fractions, decimals and percents, and apply these interrelationships in real-world situations.
- Represent numerical relationships in one- and two-dimensional graphs.
- Utilize number lines, area models, graphs, and calculator- and computer-generated numbers to represent a given quantity.

103

Standard 5: Number and Number Relationships (continued)

Student Outcomes	Examples of Indicators
Convert among percent, fractional and decimal equivalents. (I-18)	
Round off numbers to a specified place value. (I-22)	
Organize data to show relationships in tables and graphs. (IV-1)	
Interpret data from tables and charts to determine relationships. (IV-2)	
Recognize or find a specific pattern occurring in a sequence of numbers. (IV-4)	
Extend patterns and create new ones. (IV-5)	
Graph ordered pairs to show a pattern or relationship. (IV-6)	
Recognize and identify numerical and geometric patterns and sequences. (VI-1)	
Classify and categorize sets of numbers or groups of geometric shapes. (VI-2)	
Appropriately use terms such as *and, or, not, only* and *if ... then,* in a mathematical sense. (VI-4)	
Perform tasks involving inductive and deductive reasoning. (VI-7)	

Standard 6: Number Systems and Number Theory

Number systems and number theory are the foundation of the structure of mathematics. They show how limited elements integrate to form a working system.

Student Outcomes

Use concrete materials and illustrative models to represent place value in whole and decimal numbers, fractions and their equivalents and percents. (I-1)

Use appropriate symbols and order relations when comparing whole numbers, integers, common fractions and decimal fractions. (I-3)

Identify and use number properties to simplify expressions and calculations, *i.e.*, commutative, associative, distributive, identity of one and zero, rule of order of operations. (I-4)

Convert numbers in base 10 notation to and from scientific notation. (I-5)

Convert numbers in base 10 notation to and from expanded notation. (I-6)

Understand the square root of a number and the methods of finding it or its approximation. (I-7)

List the prime factorization of a whole number and ultimately write it as a numerical expression involving positive exponents. (I-8)

Understand greatest common factor and least common multiple and their applications. (I-9)

Find equivalent fractions. (I-10)

Use basic operations with whole numbers, integers, common fractions and decimal fractions. (I-12)

106

Examples of Indicators

- Demonstrate an understanding of the need for numbers beyond the whole numbers.
- Develop and use order relations for whole-number fractions and decimals.
- Identify the relationship of whole numbers as integers and fractions as rational numbers.
- Show how the basic arithmetic operations are related to one another.
- Develop and apply number theory concepts (e.g., primes, factors and multiples) in real-world and mathematical problem situations.
- Investigate the arithmetic of fractions, decimals, integers and rational numbers through the unity of common ideas.
- Develop mathematical concepts by exploring number theory.

107

Standard 6: Number Systems and Number Theory (continued)

Student Outcomes	Examples of Indicators
Use various techniques of mental arithmetic. (I-13)	
Estimate to predict answers to computational problems and to check for reasonableness of answers. (I-14)	
Use the additive and multiplicative laws of exponents. (I-16)	
Use ratios and proportions to solve problems. (I-17)	
Convert among percent, fractional and decimal equivalents. (I-18)	
Solve for the unknown in a percent problem. (I-19)	
Interpret problems, translate to numerical expressions, choose an appropriate method of calculation and solve. (I-20)	
Solve problems involving money. (I-21)	
Round off numbers to a specified place value. (I-22)	
Extend patterns and create new ones. (IV-5)	
Appropriately use terms such as *and, or, not, only* and *if ... then,* in a mathematical sense. (VI-4)	
Provide specific examples of a given numerical or geometric principle. (VI-5)	
Perform tasks involving inductive and deductive reasoning. (VI-7)	
Be familiar and proficient with the standard order of operations. (VII-2)	

Standard 7: Computation and Estimation

Computation and estimation are working skills developed through conceptual experiences. The ability to work flexibly with whole numbers, fractions, decimals, integers and rational numbers within a context of reasonableness (estimation) provides the tools necessary for all problem solving.

Student Outcomes

Identify and use number properties to simplify expressions and calculations, *i.e.*, commutative, associative, distributive, identity of one and zero, rule of order of operations. (I-4)

Convert numbers in base 10 notation to and from scientific notation. (I-5)

Convert numbers in base 10 notation to and from expanded notation. (I-6)

Understand the square root of a number and the methods of finding it or its approximation. (I-7)

List the prime factorization of a whole number and ultimately write it as a numerical expression involving positive exponents. (I-8)

Understand greatest common factor and least common multiple and their applications. (I-9)

Find equivalent fractions. (I-10)

Recall addition, subtraction, multiplication and division facts with reasonable speed. (I-11)

Use basic operations with whole numbers, integers, common fractions and decimal fractions. (I-12)

Use various techniques of mental arithmetic. (I-13)

Estimate to predict answers to computational problems and to check for reasonableness of answers. (I-14)

Examples of Indicators

- Compute with whole numbers, fractions, decimals, integers and rational numbers.
- Develop, analyze and explain procedures for computation and techniques for estimation.
- Develop, analyze and explain methods for solving proportions.
- Use the appropriate method of computation: mental arithmetic, paper-and-pencil, calculator or computer.
- Use computation, estimation and proportions to solve problems.
- Use estimation to check the reasonableness of results.
- Develop skills necessary to use appropriate technology.
- Interpret and relate information gained through the use of technology.

Standard 7: Computation and Estimation (continued)

Student Outcomes	Examples of Indicators
Use a calculator in appropriate situations. (I-15)	
Use the additive and multiplicative laws of exponents. (I-16)	
Use ratios and proportions to solve problems. (I-17)	
Convert among percent, fractional and decimal equivalents. (I-18)	
Solve for the unknown in a percent problem. (I-19)	
Interpret problems, translate to numerical expressions, choose an appropriate method of calculation and solve. (I-20)	
Solve problems involving money. (I-21)	
Round off numbers to a specified place value. (I-22)	
Add and subtract measures of time. (II-3)	
Compare temperatures and determine the amount of change. (II-4)	
Convert English and metric measurement units to equivalent units within the given system. (II-6)	
Use an appropriate formula to calculate the perimeter and area of polygons and the circumference of a circle. (II-8)	
Use an appropriate formula to calculate the volume and surface area of the following solids: spheres, prisms, pyramids, cylinders and cones. (II-9)	
Use an appropriate formula to determine a measurement when a direct measurement tool is unavailable. (II-11)	

Standard 7: Computation and Estimation (continued)

Student Outcomes	Examples of Indicators
Solve problems with measurements including distance, weight, time, area, capacity and temperature. (II-13)	
Utilize geometric principles and ideas to solve problems. (III-5)	
Experiment with, discover and apply the known formulas of geometry. (III-6)	
Understand pi and its importance in the geometry of the circle. (III-7)	
Extend patterns and create new ones. (IV-5)	
Express probability as a ratio or fraction. (V-5)	
Find the mean, median, mode and range for a set of data. (V-6)	
Appropriately use terms such as *and, or, not, only* and *if ... then,* in a mathematical sense. (VI-4)	
Provide specific examples of a given numerical or geometric principle. (VI-5)	
Perform tasks involving inductive and deductive reasoning. (VI-7)	
Be familiar and proficient with the standard order of operations. (VII-2)	
Evaluate simple algebraic expressions by substituting values for the variables. (VII-5)	
Solve two-step equations and inequalities using integers, fractions and decimals. (VII-6)	

Standard 8: Patterns and Functions

Patterns and functions are the perceptual links to connecting and understanding the world around us. Exploring patterns requires observation skills that identify, analyze, connect and extend relationships.

Student Outcomes

Recognize two- and three-dimensional figures by their shapes and identify their component parts and associated properties. (III-2)

Through observation, measurement, drawing and modeling, identify geometric properties such as symmetry, congruence, similarity, parallelism and perpendicularity. (III-3)

Experiment with, discover and apply the known formulas of geometry. (III-6)

Understand pi and its importance in the geometry of the circle. (III-7)

Recognize and classify angles as well as identify relationships between angles. (III-8)

Organize data to show relationships in tables and graphs. (IV-1)

Interpret data from tables and charts to determine relationships. (IV-2)

Determine an expression or an equation for a relationship and then evaluate or solve it for a given value of the variable. (IV-3)

Recognize or find a specific pattern occurring in a sequence of numbers. (IV-4)

Extend patterns and create new ones. (IV-5)

Graph ordered pairs to show a pattern or relationship. (IV-6)

Examples of Indicators

- Describe, extend, analyze and create a wide variety of patterns.
- Describe and represent relationships with tables, graphs and number manipulations.
- Analyze functional relationships to explain how a change in one quantity results in a change in another.
- Use patterns and functions to represent and solve problems.

Standard 8: Patterns and Functions (continued)

Student Outcomes	Examples of Indicators
Understand the relationships occurring between similar figures. (IV-9)	
Collect and organize data using lists, tables and graphs. (V-1)	
Interpret, analyze and draw conclusions from graphs. (V-2)	
Predict and record the probability of events from simple experiments. (V-4)	
Determine the probability of simple events and draw conclusions or make interpretations. (V-7)	
Find the empirical probability of an event from a sample of observed outcomes. (V-8)	
Find the probability of complementary events and of mutually exclusive events. (V-9)	
Recognize and identify numerical and geometric patterns and sequences. (VI-1)	
Classify and categorize sets of numbers or groups of geometric shapes. (VI-2)	
Appropriately use terms such as *and, or, not, only* and *if ... then,* in a mathematical sense. (VI-4)	
Perform tasks involving inductive and deductive reasoning. (VI-7)	
Represent mathematical relationships using variables. (VII-3)	

Standard 9: Algebra

Algebra is the language through which most mathematics is communicated.

Student Outcomes

Use an appropriate formula to calculate the perimeter and area of polygons and the circumference of a circle. (II-8)

Use an appropriate formula to calculate the volume and surface area of the following solids: spheres, prisms, pyramids, cylinders and cones. (II-9)

Use an appropriate formula to determine a measurement when a direct measurement tool is unavailable. (II-11)

Utilize geometric principles and ideas to solve problems. (III-5)

Experiment with, discover and apply the known formulas of geometry. (III-6)

Understand pi and its importance in the geometry of the circle. (III-7)

Organize data to show relationships in tables and graphs. (IV-1)

Interpret data from tables and charts to determine relationships. (IV-2)

Determine an expression or an equation for a relationship and then evaluate or solve it for a given value of the variable. (IV-3)

Recognize or find a specific pattern occurring in a sequence of numbers. (IV-4)

Extend patterns and create new ones. (IV-5)

Graph ordered pairs to show a pattern or relationship. (IV-6)

Examples of Indicators

- Define and use variables within the context of expressions and equations.
- Represent situations and number patterns with tables, graphs, verbal rules and equations and explore their interrelationships.
- Identify properties and relationships by using tables and graphs.
- Solve linear equations using concrete, informal and formal methods.
- Solve inequalities and nonlinear equations informally.
- Apply algebraic methods to solve a variety of real-world and mathematical problems.

Standard 9: Algebra (continued)

Student Outcomes	Examples of Indicators
Collect and organize data using lists, tables and graphs. (V-1)	
Interpret, analyze and draw conclusions from graphs. (V-2)	
Recognize and identify numerical and geometric patterns and sequences. (VI-1)	
Appropriately use terms such as *and, or, not, only* and *if ... then,* in a mathematical sense. (VI-4)	
Provide specific examples of a given numerical or geometric principle. (VI-5)	
Perform tasks involving inductive and deductive reasoning. (VI-7)	
Understand the meaning of "variable" and "constant." (VII-1)	
Be familiar and proficient with the standard order of operations. (VII-2)	
Represent mathematical relationships using variables. (VII-3)	
Simplify polynomial expressions. (VII-4)	
Evaluate simple algebraic expressions by substituting values for the variables. (VII-5)	
Solve two-step equations and inequalities using integers, fractions and decimals. (VII-6)	

122

123

Standard 10: Statistics

Collecting, representing and processing data are activities of major importance to contemporary society. Students should recognize that statistics play an important role between the exactness of mathematical studies and the nature of a world dependent largely on individual opinion.

Student Outcomes

Organize data to show relationships in tables and graphs. (IV-1)

Interpret data from tables and charts to determine relationships. (IV-2)

Determine an expression or an equation for a relationship and then evaluate or solve it for a given value of the variable. (IV-3)

Recognize or find a specific pattern occurring in a sequence of numbers. (IV-4)

Collect and organize data using lists, tables and graphs. (V-1)

Interpret, analyze and draw conclusions from graphs. (V-2)

Take polls and surveys. (V-3)

Predict and record the probability of events from simple experiments. (V-4)

Find the mean, median, mode and range for a set of data. (V-6)

Determine the probability of simple events and draw conclusions or make interpretations. (V-7)

Find the empirical probability of an event from a sample of observed outcomes. (V-8)

Find the probability of complementary events and of mutually exclusive events. (V-9)

Examples of Indicators

- Systematically collect, organize and describe data.
- Construct, read and interpret tables, charts and graphs.
- Make inferences and convincing arguments that are based on data analysis.
- Evaluate arguments that are based on data analysis.
- Formulate questions; collect and organize data based on those questions; represent the data using graphs, tables, and frequency distributions; analyze data; make conjectures; and communicate information in a meaningful way.
- Use statistical methods as a means for decision making.

Standard 10: Statistics (continued)

Student Outcomes	Examples of Indicators
Generate a frequency distribution for a given list of data. (V-10)	
Use a list or tree diagram to count permutations or combinations. (V-11)	
Appropriately use terms such as *and, or, not, only* and *if ... then,* in a mathematical sense. (VI-4)	
Perform tasks involving inductive and deductive reasoning. (VI-7)	

126

127

Standard 11: Probability

Probability is the measure of the likelihood of an event that can be determined theoretically or experimentally. Students must not only understand the relationship between the numerical expression and the probability of the events but realize that the measure of certainty or uncertainty varies as more data are collected.

Student Outcomes

Interpret data from tables and charts to determine relationships. (IV-2)

Interpret, analyze and draw conclusions from graphs. (V-2)

Predict and record the probability of events from simple experiments. (V-4)

Express probability as a ratio or fraction. (V-5)

Determine the probability of simple events and draw conclusions or make interpretations. (V-7)

Find the empirical probability of an event from a sample of observed outcomes. (V-8)

Find the probability of complementary events and of mutually exclusive events. (V-9)

Generate a frequency distribution for a given list of data. (V-10)

Use a list or tree diagram to count permutations or combinations. (V-11)

Appropriately use terms such as *and, or, not, only* and *if ... then,* in a mathematical sense. (VI-4)

Provide a counterexample of a condition that is not always true. (VI-6)

Perform tasks involving inductive and deductive reasoning. (VI-7)

Examples of Indicators

- Explore situations by experimenting and simulating probability models.
- Construct a sample to determine probability.
- Compare experimental results with mathematical expectations.
- Make predictions that are based on experimental or theoretical probabilities.
- Use charts, graphs and plots to make predictions.
- Make hypotheses, test conjectures and refine theories based on new information using experimentation and simulation.
- Explore events and situations relevant to their daily lives.

Standard 12: Geometry

Geometry is the study of spatial relationships that define and make sense of the world in which we live.

Student Outcomes

Use an appropriate formula to calculate the perimeter and area of polygons and the circumference of a circle. (II-8)

Use an appropriate formula to calculate the volume and surface area of the following solids: spheres, prisms, pyramids, cylinders and cones. (II-9)

Use a protractor to measure and draw angles. (II-10)

Use an appropriate formula to determine a measurement when a direct measurement tool is unavailable. (II-11)

Estimate lengths, areas, volumes and weights, and check by measuring in English and metric units. (II-12)

Solve problems with measurements including distance, weight, time, area, capacity and temperature. (II-13)

Use terminology appropriate to the grade level. (III-1)

Recognize two- and three-dimensional figures by their shapes, and identify their component parts and associated properties. (III-2)

Through observation, measurement, drawing and modeling, identify geometric properties such as symmetry, congruence, similarity, parallelism and perpendicularity. (III-3)

Visualize, draw and construct two- and three-dimensional figures through concrete experiences. (III-4)

Utilize geometric principles and ideas to solve problems. (III-5)

Examples of Indicators

- Identify, describe, compare and classify figures.
- Use geometric terms to describe and identify relationships.
- Define two- and three-dimensional figures through experiences in constructing, drawing, measuring, and comparing and contrasting them.
- Draw inferences and make conjectures from geometric problem situations.
- Deduce the Pythagorean theorem through explorations.
- Represent and solve problems using geometric models.
- Apply geometric principles to solve real-world problems.
- Use connections of geometry and the world through art, nature, construction and anatomy.

Standard 12: Geometry (continued)

Student Outcomes	Examples of Indicators
Experiment with, discover and apply the known formulas of geometry. (III-6)	
Understand pi and its importance in the geometry of the circle. (III-7)	
Recognize and classify angles as well as identify relationships between angles. (III-8)	
Draw and/or construct a variety of shapes having the same area. (III-9)	
Understand the relationship between angles that are complementary, supplementary and vertical. (IV-7)	
Identify and understand the relationships of angles formed by a transversal of two parallel lines. (IV-8)	
Understand the relationships occurring between similar figures. (IV-9)	
Recognize and identify numerical and geometric patterns and sequences. (VI-1)	
Classify and categorize sets of numbers or groups of geometric shapes. (VI-2)	
Using a given attribute, find similarities and differences among geometric shapes or designs. (VI-3)	
Appropriately use terms such as *and, or, not, only* and *if ... then,* in a mathematical sense. (VI-4)	
Provide specific examples of a given numerical or geometric principle. (VI-5)	
Provide a counterexample of a condition that is not always true. (VI-6)	
Perform tasks involving inductive and deductive reasoning. (VI-7)	

Standard 13: Measurement

In addition to size, distance and length of time, measurement includes the more important concept of understanding dimensional relationships regardless of the standard of measure.

Student Outcomes

Know that all measurement is approximate. (II-1)

Select the appropriate measuring instrument. (II-2)

Add and subtract measures of time. (II-3)

Compare temperatures and determine the amount of change. (II-4)

Measure length, volume and weight in both English and metric units. (II-5)

Convert English and metric measurement units to equivalent units within the given system. (II-6)

Be aware of approximate equivalent measurements between English and metric units. (II-7)

Use an appropriate formula to calculate the perimeter and area of polygons and the circumference of a circle. (II-8)

Use an appropriate formula to calculate the volume and surface area of the following solids: spheres, prisms, pyramids, cylinders and cones. (II-9)

Use a protractor to measure and draw angles. (II-10)

Use an appropriate formula to determine a measurement when a direct measurement tool is unavailable. (II-11)

Estimate lengths, areas, volumes and weights, and check by measuring in English and metric units. (II-12)

134

Examples of Indicators

- Use appropriate tools to measure objects.
- Estimate, make and use measurements to describe and compare objects.
- Select appropriate units and tools to measure the degree of accuracy required in a particular situation.
- Understand the structure and use of systems of measurement.
- Through exploration, develop procedures and formulas for determining perimeter, area, volume, angle, measure, capacity, and weight and mass.
- Develop concepts of formal and informal systems of measurements.
- Develop formulas and procedures for determining measurements when solving problems.
- Construct scale models.

135

Standard 13: Measurement (continued)

Student Outcomes	Examples of Indicators
Solve problems with measurements including distance, weight, time, area, capacity and temperature. (II-13)	
Through observation, measurement, drawing and modeling, identify geometric properties such as symmetry, congruence, similarity, parallelism and perpendicularity. (III-3)	
Visualize, draw and construct two- and three-dimensional figures through concrete experiences. (III-4)	
Utilize geometric principles and ideas to solve problems. (III-5)	
Experiment with, discover and apply the known formulas of geometry. (III-6)	
Understand pi and its importance in the geometry of the circle. (III-7)	
Recognize and classify angles as well as identify relationships between angles. (III-8)	
Draw and/or construct a variety of shapes having the same area. (III-9)	

9-12
MATHEMATICS
ESSENTIAL SKILLS

BEST COPY AVAILABLE

Arizona Mathematics Essential Skills

Mathematical Outcomes Expected of All High School Graduates

High school mathematics instruction should ensure that each student graduating from high school performs the following outcomes:

	Standards														
	1. Math as Problem Solving	2. Math as Communication	3. Math as Reasoning	4. Math Connections	5. Algebra	6. Functions	7. Geometry from a Synthetic	8. Geometry from an Algebraic	9. Trigonometry	10. Statistics	11. Probability	12. Discrete Math	13. Conceptual Underpinnings of Calc.	14. Math Structure	
I. Number															
1. Use the four arithmetic operations on rational numbers (expressed as common fractions, decimal fractions or percents) with accuracy and reasonable speed, using pencil and paper or calculator.	X	X	X	X										X	1
2. Perform mental calculations of the four operations with appropriate levels of precision.	X	X	X	X										X	2
3. Understand the concept of ordering the real numbers by locating their corresponding points on the number line.	X	X	X	X										X	3
4. Write mathematical expressions and sentences to replace verbal expressions, using appropriate operation, number and relation symbols.	X	X	X	X	X										4
5. Use standard rules for order of operations, together with symbols of inclusion to correctly evaluate numerical expressions and formulas in a variety of problem settings.	X	X	X	X										X	5
6. Understand and use properties of equality and inequality.	X	X	X	X										X	6
II. Measurement															
1. Understand the approximate nature of measurement and apply the precision of measurements to the resulting accuracy of calculations.	X	X	X	X										X	1
2. Make unit conversions with appropriate levels of accuracy between and within standard systems of measures.	X	X	X	X										X	2
3. Use appropriate standard units of measurement to make reasonable estimates of linear, area, volume and weight measures of objects commonly encountered in daily life.	X	X	X	X	X		X								3

Arizona Mathematics Essential Skills (continued)

	Standards														
	1. Math as Problem Solving	2. Math as Communication	3. Math as Reasoning	4. Math Connections	5. Algebra	6. Functions	7. Geometry from a Synthetic	8. Geometry from an Algebraic	9. Trigonometry	10. Statistics	11. Probability	12. Discrete Math	13. Conceptual Underpinnings of Calc.	14. Math Structure	
II. Measurement (continued)															
4. Select and use appropriate formulas or procedures to determine indirectly a measure when a direct measurement is not available or feasible.	X	X	X	X	X										4
III. Geometry															
1. Identify and distinguish between geometric elements, such as lines, angles, planes, polygons, circles and regular solids.	X	X	X	X			X								1
2. Understand and apply basic geometric relationships such as parallel, intersecting, perpendicular, similarity, proportionality and congruence.	X	X	X	X			X								2
3. Calculate areas of regular polygons and circles and volumes of rectangular solids, cylinders and spheres.	X	X	X	X			X	X							3
4. Use compass, protractor and ruler to perform standard geometric constructions.	X	X	X	X			X								4
5. Apply the Pythagorean theorem in solving problems.	X	X	X	X			X								5
IV. Data Analysis and Probability															
1. Select and use appropriate principles of counting collections and arrangements of objects or outcomes of sequential procedures.	X	X	X	X						X	X				1
2. Select and use appropriate statistical measures to describe sets of data.	X	X	X	X						X					2

Arizona Mathematics Essential Skills (continued)

	Standards														
	1. Math as Problem Solving	2. Math as Communication	3. Math as Reasoning	4. Math Connections	5. Algebra	6. Functions	7. Geometry from a Synthetic	8. Geometry from an Algebraic	9. Trigonometry	10. Statistics	11. Probability	12. Discrete Math	13. Conceptual Underpinnings of Calc.	14. Math Structure	
IV. Data Analysis and Probability (continued)															
3. Explain variability of statistical measures in terms of sampling process or population differences.	X	X	X	X		X				X					3
4. Use the concept of mathematical expectation to estimate outcomes of random processes.	X	X	X	X							X				4
5. Identify and explain misuses of statistics.	X	X	X	X						X					5
6. Use experimental observations to estimate empirical probabilities and population parameters.	X	X	X	X							X				6
7. Distinguish between independent and dependent events and use conditional probabilities.	X	X	X	X							X				7
V. Algebra															
1. Add, subtract, multiply and divide with monomials and binomials.	X	X	X	X	X										1
2. Simplify and evaluate algebraic expressions which involve integral exponents and square roots.	X	X	X	X	X										2
3. Simplify rational algebraic expressions having monomial denominators.	X	X	X	X	X										3
4. Solve linear equations and inequalities in one variable, and apply these to solve problems.	X	X	X	X	X										4
5. Solve systems of linear equations and inequalities in two variables, graphically and analytically, and formulate such systems to solve problems.	X	X	X	X	X										5
6. Solve problems using ratio and proportion, percent or direct and inverse variation.	X	X	X	X	X										6

Arizona Mathematics Essential Skills (continued)

	Standards														
	1. Math as Problem Solving	2. Math as Communication	3. Math as Reasoning	4. Math Connections	5. Algebra	6. Functions	7. Geometry from a Synthetic	8. Geometry from an Algebraic	9. Trigonometry	10. Statistics	11. Probability	12. Discrete Math	13. Conceptual Underpinnings of Calc.	14. Math Structure	
VI. Patterns and Relations															
1. Identify and distinguish between arithmetic and geometric progressions and state a rule or formula for the general term of such a progression.	X	X	X	X	X		X								1
2. Write the linear relationship between two variables by inspection of its graph or of the set of its ordered pairs.	X	X	X	X	X										2
3. Graph the inverse of a function by inspection of the graph of the function.	X	X	X	X	X										3
4. Identify and explain graphic misrepresentation or distortions of sets of data.	X	X	X	X	X					X					4
5. Identify, interpret and construct graphs of nonlinear relations such as parabolas and circles.	X	X	X	X	X	X			X						5
VII. Analytic Reasoning															
1. Distinguish between inductive and deductive reasoning and explain how each is most appropriately used.	X	X	X	X		X	X								1
2. Recognize when conditions of definitions are satisfied.	X	X	X	X	X		X								2
3. Use deductive reasoning to generate conclusions.	X	X	X	X			X								3
4. Use statistical reasoning to test hypotheses informally.	X	X	X	X						X					4
5. Identify valid and invalid arguments.	X	X	X	X			X			X					5

Arizona Mathematics Essential Skills (continued)

Additional Courses and Skills Required for University Admission

Upon completing courses in Algebra I, Algebra II and Geometry, a high school student should have mastered the following outcomes:

I. Algebra I

	Standards														
	1. Math as Problem Solving	2. Math as Communication	3. Math as Reasoning	4. Math Connections	5. Algebra	6. Functions	7. Geometry from a Synthetic	8. Geometry from an Algebraic	9. Trigonometry	10. Statistics	11. Probability	12. Discrete Math	13. Conceptual Underpinnings of Calc.	14. Math Structure	
1. Simplify algebraic expressions.	X	X	X	X	X										1
2. Evaluate algebraic expressions.	X	X	X	X	X										2
3. Solve linear equations.	X	X	X	X	X										3
4. Use the laws of exponents to simplify expressions involving integral exponents.	X	X	X	X	X										4
5. Add, subtract, multiply and divide polynomial expressions.	X	X	X	X	X										5
6. Factor polynomials.	X	X	X	X	X										6
7. Solve linear inequalities and graph them on a number line.	X	X	X	X	X										7
8. Find the slope of a line and write equations of lines in standard and slope intercept form.	X	X	X	X	X	X									8
9. Solve systems of linear equations in two variables by substitution, addition method and graphically.	X	X	X	X	X										9
10. Add, subtract, multiply and divide algebraic fractions and complex fractions.	X	X	X	X	X										10

Arizona Mathematics Essential Skills (continued)

	Standards														
	1. Math as Problem Solving	2. Math as Communication	3. Math as Reasoning	4. Math Connections	5. Algebra	6. Functions	7. Geometry from a Synthetic	8. Geometry from an Algebraic	9. Trigonometry	10. Statistics	11. Probability	12. Discrete Math	13. Conceptual Underpinnings of Calc.	14. Math Structure	
I. Algebra I (continued)															
11. Simplify, add, subtract and multiply radicals, and rationalize denominators.	X	X	X	X	X	X									11
12. Solve direct and inverse variation problems.	X	X	X	X	X										12
13. Solve elementary word problems.	X	X	X	X	X										13
14. Use formulas.	X	X	X	X	X										14
15. Solve and graph absolute value problems.	X	X	X	X	X										15
16. Solve quadratic equations.	X	X	X	X	X										16
17. Understand the concept of functions and their graphs.	X	X	X	X	X	X									17
II. Algebra II															
1. Evaluate algebraic expressions.	X	X	X	X	X										1
2. Use laws of exponents to change the forms of expressions involving integral exponents and rational exponents.	X	X	X	X	X										2
3. Simplify radicals.	X	X	X	X	X										3
4. Change from fractional exponents to radical form or radical form to fractional exponents whenever necessary.	X	X	X	X	X										4
5. Solve formulas for specified symbols.	X	X	X	X	X										5

150

151

Arizona Mathematics Essential Skills (continued)

II. Algebra II (continued)

	Standards														
	1. Math as Problem Solving	2. Math as Communication	3. Math as Reasoning	4. Math Connections	5. Algebra	6. Functions	7. Geometry from a Synthetic	8. Geometry from an Algebraic	9. Trigonometry	10. Statistics	11. Probability	12. Discrete Math	13. Conceptual Underpinnings of Calc.	14. Math Structure	
6. Solve equations and inequalities involving absolute value.	X	X	X	X	X										6
7. Perform operations with polynomials including synthetic division.	X	X	X	X	X										7
8. Factor various types of polynomials.	X	X	X	X	X										8
9. Solve quadratic equations by factoring and using the quadratic formula.	X	X	X	X	X										9
10. Solve quadratic inequalities.	X	X	X	X	X										10
11. Perform operations with rational expressions.	X	X	X	X	X										11
12. Solve equations and inequalities involving rational expressions.	X	X	X	X	X										12
13. Perform operations with complex numbers.	X	X	X	X	X	X									13
14. Work with forms of the equation of a line and the concept of the slope of a line.	X	X	X	X	X	X									14
15. Solve linear systems of equations in three variables.	X	X	X	X	X										15
16. Work with the function concept and function notation.	X	X	X	X	X	X									16
17. Use the method of completing the square to find the vertex of a parabola and then graph.	X	X	X	X	X										17
18. Solve exponential and logarithmic equations.	X	X	X	X	X										18

Arizona Mathematics Essential Skills (continued)

	Standards														
	1. Math as Problem Solving	2. Math as Communication	3. Math as Reasoning	4. Math Connections	5. Algebra	6. Functions	7. Geometry from a Synthetic	8. Geometry from an Algebraic	9. Trigonometry	10. Statistics	11. Probability	12. Discrete Math	13. Conceptual Underpinnings of Calc.	14. Math Structure	
II. Algebra II (continued)															
19. Work with arithmetic and geometric progressions.	X	X	X	X	X										19
20. Use a calculator to obtain two-decimal-place approximations to the answer for radical, exponential or logarithmic forms.	X	X	X	X	X										20
21. Work with conic sections, and use algebra to solve problems involving coordinate geometry.	X	X	X	X	X			X							21
22. Solve equations and problems which rely on properties of logarithmic and exponential functions.	X	X	X	X	X										22
23. Understand degree and radian measure of angles.	X	X	X	X			X		X						23
24. Work with trigonometric functions as ratios of sides of right triangles and as circular functions.	X	X	X	X			X		X						24
25. Graph trigonometric functions.	X	X	X	X					X						25
26. Use calculators and tables to approximate values of trigonometric functions.	X	X	X	X			X		X						26
27. Derive exact values of trigonometric functions associated with angles of 30, 45, 60 and 90 degrees.	X	X	X	X			X		X						27
28. Describe and graph one-to-one functions and their inverses.	X	X	X	X	X										28
29. Work with inverse trigonometric functions.	X	X	X	X	X				X						29
30. Verify trigonometric identities and solve trigonometric equations.	X	X	X	X	X				X						30
31. Solve right triangles and use the law of sines and the law of cosines.	X	X	X	X			X		X						31

Arizona Mathematics Essential Skills (continued)

III. Geometry

	Standards														
	1. Math as Problem Solving	2. Math as Communication	3. Math as Reasoning	4. Math Connections	5. Algebra	6. Functions	7. Geometry from a Synthetic	8. Geometry from an Algebraic	9. Trigonometry	10. Statistics	11. Probability	12. Discrete Math	13. Conceptual Underpinnings of Calc.	14. Math Structure	
1. Use definitions, postulates and theorems to develop proofs.	X	X	X	X			X								1
2. Solve word problems.	X	X	X	X			X	X							2
3. Perform algebraic manipulations.	X	X	X	X				X							3
4. Use formal, logical reasoning to solve abstract, theoretical and practical problems.	X	X	X	X			X								4
5. Understand angle and line relationships, *e.g.*, vertical angles, parallel or perpendicular lines, and supplementary and complementary angles.	X	X	X	X			X								5
6. Use concepts of congruence and similarity.	X	X	X	X		X	X								6
7. Explore polygonal relationships to include and identify common geometric shapes and describe their angle measures.	X	X	X	X			X								7
8. Explore circles and arc characteristics to determine the measure of arcs and their related angles and chords, tangents and secants.	X	X	X	X			X								8
9. Know and use the Pythagorean theorem.	X	X	X	X			X								9
10. Find the perimeter and area of a plane figure.	X	X	X	X			X	X							10
11. Find volume and surface area of a solid.	X	X	X	X			X	X							11

BEST COPY AVAILABLE

Arizona Mathematics Essential Skills (continued)

III. Geometry (continued)

	Standards														
	1. Math as Problem Solving	2. Math as Communication	3. Math as Reasoning	4. Math Connections	5. Algebra	6. Functions	7. Geometry from a Synthetic	8. Geometry from an Algebraic	9. Trigonometry	10. Statistics	11. Probability	12. Discrete Math	13. Conceptual Underpinnings of Calc.	14. Math Structure	
12. Make and apply geometric construction.	X	X	X	X			X								12
13. Solve problems applying the methods of coordinate geometry, *e.g.*, the distance formula, the midpoint formula and finding the equation of a line.	X	X	X	X				X							13
14. Use right triangle trigonometry to solve problems.	X	X	X	X			X		X						14

158

159

Standard 1: Mathematics as Problem Solving

Problem solving is mathematics in its broadest sense. In the primary grades it is sometimes useful to differentiate among conceptual, procedural and problem-solving goals. In grades 9-12 the distinctions between these goals blur, and they are integrated and internalized. This gives the student a broad foundation for approaching and doing mathematics, regardless of topic or subject. Problem solving is more than applying a specific rule to a specific problem. Problem solving is the fabric that holds the state Essential Skills, *NCTM Standards* and all of mathematics together.

Student Outcomes

Apply the mathematics Essential Skills in a variety of problem-solving contexts and learn the more complex Essential Skills within the context of real-life problems or situations.

160

Examples of Indicators

- Use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content.
- Apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics.
- Recognize and formulate problems from situations within and outside mathematics.
- Apply the process of mathematical modeling to real-world problem situations.

161

Standard 2: Mathematics as Communications

All students must listen to, read about, write about, speak about, reflect on and demonstrate mathematical ideas. Students involved in individual and small-group activities will have many opportunities to discuss, question, listen and summarize. These activities will direct instruction away from recall of terminology and routine manipulation to deeper conceptual understanding of mathematics. It is not enough for students to find **the** answer. It is just as important for students to describe how they get their answer and the difficulties they encounter in trying to find it. It has become more apparent, with the development of the Arizona Student Assessment Program, that mathematics is not just calculations and manipulations. Discussions about people, issues and cultural implications of mathematics will add to the understanding of the connection between mathematics and society.

Student Outcomes

Communicate their experiences with, and an understanding of, the mathematics Essential Skills.

Examples of Indicators

- Reflect upon and clarify thinking about mathematical ideas and relationships.
- Formulate mathematical definitions and express generalizations discovered through investigations.
- Express mathematical ideas orally and in writing.
- Demonstrate understanding of written presentations of mathematics.
- Ask clarifying and extending questions related to mathematics that apply to the real world.

Standard 3: Mathematics as Reasoning

Deductive and inductive reasoning are used by themselves in all areas of mathematics. While doing mathematics, students make conjectures from patterns made in particular cases (inductive reasoning) and test these conjectures by logical verification, or a counterexample (deductive reasoning). These methods are very useful in mathematics and in other fields. All students need to know that deductive reasoning is the method used to establish the validity of a mathematical assertion. Reasoning and proofs need to be used in all mathematics courses.

Student Outcomes

Use mathematical reasoning to demonstrate conceptual understanding of the mathematics Essential Skills and their ability to use them in a variety of problem situations.

164

Examples of Indicators

- Make and test conjectures.
- Formulate counterexamples.
- Judge the validity of arguments.
- Construct simple valid arguments.

And so that, in addition, college-intending students—

- Construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

165

Standard 4: Mathematical Connections

Connections are the threads that take isolated skills and weave them into useful tools. Connections need to be cultivated between mathematical topics, as well as with other disciplines.

Developing mathematics as an integrated whole increases the potential for retention and transfer of mathematical concepts. Connecting mathematics with other disciplines and with daily affairs underscores the utility of the subject.

Student Outcomes

Students can make connections between the different mathematics Essential Skills, as well as with real-life situations.

Examples of Indicators

- Recognize equivalent representations of the same concept.
- Relate procedures in one representation to procedures in an equivalent representation.
- Use the connections among mathematical topics.
- Use the connections between mathematics and other disciplines and the real world.

  Art: the use of symmetry, perspective, spatial representations, and patterns (including fractals) to create original artistic works

  Biology: the use of scaling to identify limiting factors on the organisms

  Business: the optimization of a communication network

  Industrial Arts: the use of mathematics-based computer-aided design in producing scale drawings or models of three-dimensional objects such as houses

  Medicine: modeling an inoculation plan to eliminate an infectious disease

  Physics: the use of vectors to address problems involving forces

  Social Science: the use of statistical techniques in predicting and analyzing election results

Standard 5: Algebra

Algebra is the language through which most of mathematics is communicated. The increasing use of quantitative methods, both in natural sciences and in disciplines such as economics, psychology and sociology, has made algebra an important tool for applied mathematics.

Student Outcomes

Write mathematical expressions and sentences to replace verbal expressions, using appropriate operation, number and relation symbols. (I-4)

Use appropriate standard units of measurement to make reasonable estimates of linear, area, volume and weight measures of objects commonly encountered in daily life. (II-3)

Select and use appropriate formulas or procedures to determine indirectly a measure when a direct measurement is not available or feasible. (II-4)

Add, subtract, multiply and divide with monomials and binomials. (V-1)

Simplify and evaluate algebraic expressions which involve integral exponents and square roots. (V-2)

Simplify rational algebraic expressions having monomial denominators. (V-3)

Solve linear equations and inequalities in one variable, and apply these to solve problems. (V-4)

Solve systems of linear equations and inequalities in two variables, graphically and analytically, and formulate such systems to solve problems. (V-5)

Solve problems using ratio and proportion, percent or direct and inverse variation. (V-6)

Identify and distinguish between arithmetic and geometric progressions and state a rule or formula for the general term of such a progression. (VI-1)

Examples of Indicators

- Represent situations that involve variable quantities with expressions, equations, inequalities and matrices.
- Use tables and graphs as tools to interpret expressions, equations and inequalities.
- Operate on expressions and matrices, and solve equations and inequalities.
- Transfer the algebraic process to problem solving in other disciplines.

Standard 5: Algebra (continued)

Student Outcomes	Examples of Indicators
Write the linear relationship between two variables by inspection of its graph or of the set of its ordered pairs. (VI-2)	
Graph the inverse of a function by inspection of the graph of the function. (VI-3)	
Identify and explain graphic misrepresentation or distortions of sets of data. (VI-4)	
Identify, interpret and construct graphs of nonlinear relations such as parabolas and circles. (VI-5)	
Recognize when conditions of definitions are satisfied. (VII-2)	
Additional Courses and Skills Required for University Admission	**And so that, in addition, college-intending students can—**
Upon completing courses in Algebra I, Algebra II and Geometry, a high school student should have mastered the following outcomes:	◆ Use matrices to solve linear systems.
Simplify algebraic expressions. (I-1)	◆ Demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations.
Evaluate algebraic expressions. (I-2)	
Solve linear equations. (I-3)	
Use the laws of exponents to simplify expressions involving integral exponents. (I-4)	

Standard 5: Algebra (continued)

Student Outcomes	Examples of Indicators
Add, subtract, multiply and divide polynomial expressions. (I-5)	
Factor polynomials. (I-6)	
Solve linear inequalities and graph them on a number line. (I-7)	
Find the slope of a line and write equations of lines in standard and slope intercept form. (I-8)	
Solve systems of linear equations in two variables by substitution, addition method and graphically. (I-9)	
Add, subtract, multiply and divide algebraic fractions and complex fractions. (I-10)	
Simplify, add, subtract and multiply radicals, and rationalize denominators. (I-11)	
Solve direct and inverse variation problems. (I-12)	
Solve elementary word problems. (I-13)	
Use formulas. (I-14)	
Solve and graph absolute value problems. (I-15)	
Solve quadratic equations. (I-16)	
Understand the concept of functions and their graphs. (I-17)	
Evaluate algebraic expressions. (II-1)	

Standard 5: Algebra (continued)

Student Outcomes	Examples of Indicators
Use laws of exponents to change the forms of expressions involving integral exponents and rational exponents. (II-2)	
Simplify radicals. (II-3)	
Change from fractional exponents to radical form or radical form to fractional exponents whenever necessary. (II-4)	
Solve formulas for specified symbols. (II-5)	
Solve equations and inequalities involving absolute value. (II-6)	
Perform operations with polynomials including synthetic division. (II-7)	
Factor various types of polynomials. (II-8)	
Solve quadratic equations by factoring and using the quadratic formula. (II-9)	
Solve quadratic inequalities. (II-10)	
Perform operations with rational expressions. (II-11)	
Solve equations and inequalities involving rational expressions. (II-12)	
Perform operations with complex numbers. (II-13)	
Work with forms of the equation of a line and the concept of the slope of a line. (II-14)	
Solve linear systems of equations in three variables. (II-15)	

Standard 5: Algebra (continued)

Student Outcomes	Examples of Indicators
Work with the function concept and function notation. (II-16)	
Use the method of completing the square to find the vertex of a parabola and then graph. (II-17)	
Solve exponential and logarithmic equations. (II-18)	
Work with arithmetic and geometric progressions. (II-19)	
Use a calculator to obtain two-decimal-place approximations to the answer for radical, exponential or logarithmic forms. (II-20)	
Work with conic sections, and use algebra to solve problems involving coordinate geometry. (II-21)	
Solve equations and problems which rely on properties of logarithmic and exponential functions. (II-22)	
Describe and graph one-to-one functions and their inverses. (II-28)	
Work with inverse trigonometric functions. (II-29)	
Verify trigonometric identities and solve trigonometric equations. (II-30)	

Standard 6: Functions

Functions are special correspondences between the elements of two sets in a given field of mathematics. They are reclusive within a given field but may occur as operations in other mathematical areas. The function concept is important because it is a mathematical representation of many input-output situations found in the real world, including those that recently have arisen as a result of technological advances.

Student Outcomes

Explain variability of statistical measures in terms of sampling process or population differences. (IV-3)

Identify, interpret and construct graphs of nonlinear relations such as parabolas and circles. (VI-5)

Distinguish between inductive and deductive reasoning and explain how each is most appropriately used. (VII-1)

Additional Courses and Skills Required for University Admission

Upon completing courses in Algebra I, Algebra II and Geometry, a high school student should have mastered the following outcomes:

Find the slope of a line and write equations of lines in standard and slope intercept form. (I-8)

Simplify, add, subtract and multiply radicals, and rationalize denominators. (I-11)

Understand the concept of functions and their graphs. (I-17)

Perform operations with complex numbers. (II-13)

Work with forms of the equation of a line and the concept of the slope of a line. (II-14)

Work with the function concept and function notation. (II-16)

Use concepts of congruence and similarity. (III-6)

Examples of Indicators

- Model real-world problems with a variety of functions.
- Represent and analyze relationships using tables, verbal rules, equations and graphs.
- Translate among tabular, symbolic and graphical representations of functions.
- Demonstrate that a variety of problem situations can be modeled by the same type of function.
- Determine the effects of parameter changes on the graphs of functions.

And so that, in addition, college-intending students can—

- Perform operations on, and the general properties and behavior of, classes of functions.

Standard 7: Geometry from a Synthetic Perspective

Geometry provides experiences that deepen students' understanding of shapes and their properties, with an emphasis on their wide applicability in human activity. Physical models, three-dimensional figures and other real-world objects should be used to provide a strong base for the development of geometric intuition that will be used in working with abstract ideas.

Student Outcomes

Use appropriate standard units of measurement to make reasonable estimates of linear, area, volume and weight measures of objects commonly encountered in daily life. (II-3)

Identify and distinguish between geometric elements, such as lines, angles, planes, polygons, circles and regular solids. (III-1)

Understand and apply basic geometric relationships such as parallel, intersecting, perpendicular, similarity, proportionality and congruence. (III-2)

Calculate areas of regular polygons and circles and volumes of rectangular solids, cylinders and spheres. (III-3)

Use compass, protractor and ruler to perform standard geometric constructions. (III-4)

Apply the Pythagorean theorem in solving problems. (III-5)

Identify and distinguish between arithmetic and geometric progressions and state a rule or formula for the general term of such a progression. (VI-1)

Distinguish between inductive and deductive reasoning and explain how each is most appropriately used. (VII-1)

Recognize when conditions of definitions are satisfied. (VII-2)

Use deductive reasoning to generate conclusions. (VII-3)

Examples of Indicators

- Interpret and draw three-dimensional objects.
- Represent problem situations with geometric models and apply properties of figures.
- Classify figures in terms of congruence and similarity and apply these relationships.
- Deduce properties of, and relationships between, figures from given assumptions.

Standard 7: Geometry from a Synthetic Perspective (continued)

Student Outcomes

Identify valid and invalid arguments. (VII-5)

Additional Courses and Skills Required for University Admission

Upon completing courses in Algebra I, Algebra II and Geometry, a high school student should have mastered the following outcomes:

Understand degree and radian measure of angles. (II-23)

Work with trigonometric functions as ratios of sides of right triangles and as circular functions. (II-24)

Use calculators and tables to approximate values of trigonometric functions. (II-26)

Derive exact values of trigonometric functions associated with angles of 30, 45, 60 and 90 degrees. (II-27)

Solve right triangles and use the law of sines and the law of cosines. (II-31)

Use definitions, postulates and theorems to develop proofs. (III-1)

Solve word problems. (III-2)

Use formal, logical reasoning to solve abstract, theoretical and practical problems. (III-4)

Understand angle and line relationships, *e.g.*, vertical angles, parallel or perpendicular lines, and supplementary and complementary angles. (III-5)

Examples of Indicators

And so that, in addition, college-intending students can—

- Develop an understanding of an axiomatic system through investigating and comparing various geometries.

Standard 7: Geometry from a Synthetic Perspective (continued)

Student Outcomes	Examples of Indicators
Use concepts of congruence and similarity. (III-6)	
Explore polygonal relationships to include and identify common geometric shapes and describe their angle measures. (III-7)	
Explore circles and arc characteristics to determine the measure of arcs and their related angles and chords, tangents, and secants. (III-8)	
Know and use the Pythagorean theorem. (III-9)	
Find the perimeter and area of a plane figure. (III-10)	
Find volume and surface area of a solid. (III-11)	
Make and apply geometric construction. (III-12)	
Use right triangle trigonometry to solve problems. (III-14)	

Standard 8: Geometry from an Algebraic Perspective

One of the most important connections in all of mathematics is that between geometry and algebra. The interplay between geometry and algebra strengthens students' ability to formulate and analyze problems from a variety of perspectives.

Student Outcomes

Calculate areas of regular polygons and circles and volumes of rectangular solids, cylinders and spheres. (III-3)

Additional Courses and Skills Required for University Admission

Upon completing courses in Algebra I, Algebra II and Geometry, a high school student should have mastered the following outcomes:

Work with conic sections, and use algebra to solve problems involving coordinate geometry. (II-21)

Solve word problems. (III-2)

Perform algebraic manipulations. (III-3)

Find the perimeter and area of a plane figure. (III-10)

Find volume and surface area of a solid. (III-11)

Solve problems applying the methods of coordinate geometry, *e.g.*, the distance formula, the midpoint formula and finding the equation of a line. (III-13)

Examples of Indicators

- Translate between synthetic and coordinate representations.
- Determine properties of figures using transformations and using coordinates.
- Identify congruent and similar figures using transformations.
- Analyze properties of Euclidean transformations and relate translations to vectors.

And so that, in addition, college-intending students can—

- Determine properties of figures using vectors.
- Use transformations, coordinates and vectors in problem solving.

Standard 9: Trigonometry

Trigonometry is a study of triangular relationships and circular functions. Its many real-world applications include navigation, surveying, architecture and engineering, through which it harmonizes geometry and algebra.

Student Outcomes

Identify, interpret and construct graphs of nonlinear relations such as parabolas and circles. (VI-5)

Additional Courses and Skills Required for University Admission

Upon completing courses in Algebra I, Algebra II and Geometry, a high school student should have mastered the following outcomes:

Understand degree and radian measure of angles. (II-23)

Work with trigonometric functions as ratios of sides of right triangles and as circular functions. (II-24)

Graph trigonometric functions. (II-25)

Use calculators and tables to approximate values of trigonometric functions. (II-26)

Derive exact values of trigonometric functions associated with angles of 30, 45, 60 and 90 degrees. (II-27)

Work with inverse trigonometric functions. (II-29)

Verify trigonometric identities and solve trigonometric equations. (II-30)

Solve right triangles and use the law of sines and the law of cosines. (II-31)

Use right triangle trigonometry to solve problems. (III-14)

Examples of Indicators

- Apply trigonometry to solve problems involving triangles.
- Represent periodic real-world situations using the sine and cosine functions.

And so that, in addition, college-intending students can —

- Show the connection between triangular and circular trigonometric functions.
- Use circular functions to model periodic real-world situations.
- Graph trigonometric functions.
- Solve trigonometric equations and verify trigonometric identities.
- Demonstrate the connections between trigonometric functions and polar coordinates, complex numbers and series.

Standard 10: Statistics

Collecting, representing and processing data are activities of major importance to contemporary society. Students should recognize that statistics play an important intermediate role between the exactness of mathematical studies and the equivocal nature of a world dependent largely on individual opinion.

Student Outcomes

Select and use appropriate principles of counting collections and arrangements of objects or outcomes of sequential procedures. (IV-1)

Select and use appropriate statistical measures to describe sets of data. (IV-2)

Explain variability of statistical measures in terms of sampling process or population differences. (IV-3)

Identify and explain misuses of statistics. (IV-5)

Identify and explain graphic misrepresentation or distortions of sets of data. (VI-4)

Use statistical reasoning to test hypotheses informally. (VII-4)

Identify valid and invalid arguments. (VII-5)

Examples of Indicators

- Construct and draw inferences from charts, tables and graphs that summarize data from real-world situations.
- Use curve fitting to predict from data.
- Apply measures of central tendency, variability and correlation.
- Use sampling to explain its role in statistical claims.
- Design a statistical experiment to study a problem, conduct the experiment and interpret and communicate the outcomes.
- Explain the effects of data transformations on measures of central tendency and variability.

And so that, in addition, college-intending students can—

- Transform data to aid in data interpretation and prediction.
- Test hypotheses using appropriate statistics.

Standard 11: Probability

Probability provides concepts and methods for dealing with uncertainty and for interpreting predictions based on uncertainty. Its study provides a student with a basis of understanding from which to make informed observations about the likelihood of events and to interpret the validity of statistical claims.

Student Outcomes

Select and use appropriate principles of counting collections and arrangements of objects or outcomes of sequential procedures. (IV-1)

Use the concept of mathematical expectation to estimate outcomes of random processes. (IV-4)

Use experimental observations to estimate empirical probabilities and population parameters. (IV-6)

Distinguish between independent and dependent events and use conditional probabilities. (IV-7)

Examples of Indicators

- Use experimental or theoretical probability, as appropriate, to represent and solve problems involving uncertainty.
- Use simulations to estimate probabilities.
- Explain the concept of a random variable.
- Create and interpret discrete probability distributions.
- Describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed.

And so that, in addition, college-intending students can—

- Use the concept of a random variable to generate and interpret probability distributions including binomial, uniform, normal and chi square.

Standard 12: Discrete Mathematics

The nonmaterial world of information processing requires the use of discrete (discontinuous) mathematics. Computers are essentially finite, discrete machines, and the topics from discrete mathematics are essential to solving problems using computer methods. Discrete mathematics is the field of mathematical properties of sets and systems that have only a finite number of elements.

Student Outcomes

Standard 12: Discrete Mathematics is an NCTM standard that is not currently addressed in the Arizona Essential Skills. When the Essential Skills are revised and the importance of this area increases, it will be addressed.

Examples of Indicators

- Represent problem situations using discrete structures such as finite graphs, matrices, sequences and recurrence relations.
- Represent and analyze finite graphs using matrices.
- Develop and analyze algorithms.
- Solve enumeration and finite probability problems.

And so that, in addition, college-intending students can—

- Represent and solve problems using linear programming and difference equations.
- Investigate problem situations that arise in connection with computer validation and the application of algorithms.

Standard 13: Conceptual Underpinning of Calculus

This standard calls for the opportunity for students to systematically, but informally, investigate the central ideas of calculus: limit, the area under a curve, the rate of change and the slope of a tangent line. This standard will extend students' knowledge of function characteristics and introduce them to the mode of infinite processes. Such a study will deepen a student's understanding of function and its utility in representing and answering questions about real-world situations.

Student Outcomes

Standard 13: Conceptual Underpinning of Calculus is an NCTM standard that is not currently addressed in the Arizona Essential Skills. When the Essential Skills are revised and the importance of this area increases, the conceptual underpinnings of calculus will be addressed.

196

Examples of Indicators

- Determine maximum and minimum points of a graph and interpret the results in problem situations.
- Investigate limiting processes by examining infinite sequences, series and areas under curves.

And so that, in addition, college-intending students can—

- Understand the conceptual foundations of limit, the area under a curve, the rate of change, the slope of a tangent line and the applications of each in other disciplines.
- Analyze the graphs of polynomial, rational, radical and transcendental functions.

197

Standard 14: Mathematical Structure

The structure of mathematics is the framework upon which its various fields are built. Descarte called mathematics the "science of order." Students should become aware of this structure, how it provides a strong foundation on which a variety of content strands are built and how it simultaneously holds these different strands together. Each field offers its own pattern of order. As a science of patterns, mathematics is a mode of inquiry that reveals fundamental truth about the order of our world. An awareness of these broad structuring principles frees students to take a more constructive approach to new mathematical topics. The degree of formalism must be consistent with the student's level of mathematical maturity.

Student Outcomes

Use the four arithmetic operations on rational numbers (expressed as common fractions, decimal fractions or percents) with accuracy and reasonable speed, using pencil and paper or calculator. (I-1)

Perform mental calculations of the four operations with appropriate levels of precision. (I-2)

Understand the concept of ordering the real numbers by locating their corresponding points on the number line. (I-3)

Use standard rules for order of operations together with symbols of inclusion to correctly evaluate numerical expressions and formulas in a variety of problem settings. (I-5)

Understand and use properties of equality and inequality. (I-6)

Understand the approximate nature of measurement and apply the precision of measurements to the resulting accuracy of calculations. (II-1)

Make unit conversions with appropriate levels of accuracy between and within standard systems of measures. (II-2)

Examples of Indicators

- Compare and contrast the real number system and its various subsystems with regard to their structural characteristics.
- Apply the logic of algebraic procedures.
- Compare seemingly different mathematical systems to determine if they are essentially the same.

And so that, in addition, college-intending students can—

- Develop the complex number system and demonstrate facility with its operations.
- Prove elementary theorems within various mathematical structures, such as groups and fields.
- Explain the nature and purpose of axiomatic systems.

GOAL 3. *Integrate Student Assessment with the Learning Process*

The most accurate reflection of students' progress in mathematics is evidenced by the students' performance in the classroom. In order for assessment to be valid it must be integrated into the learning process. The integration of assessment and learning in Arizona schools is being strongly impacted by the Arizona Student Assessment Program and the District Assessment Plan.

Arizona Student Assessment Program (ASAP)

Working as partners, the Joint Legislative Committee on Goals for Educational Excellence, the state Board of Education and the Arizona Department of Education established goals for improving K-12 student achievement. The committee adopted the state Board-approved mathematics Essential Skills as the high standard for achievement in Arizona. The Arizona Student Assessment Program was then developed to assess Arizona's student progress in mastering the Essential Skills. The mathematics assessments gave depth to the Essential Skills and a new way of looking at and thinking about curriculum, instruction and assessment. But the Arizona mathematics Essential Skills were interpreted by many districts as a list of isolated skills and were assessed accordingly. Therefore, the 1987 document was reformatted to illustrate that the mathematics Essential Skills are multifaceted and aligned with the ASAP mathematics assessments in both content and process.

The Arizona Student Assessment Program, a comprehensive statewide program, was initiated to raise the standards for curriculum, instruction and assessment. It sets world-class standards in mathematics. It is evident that the traditional standardized tests alone, which measure student against student on isolated skills once a year, will not suffice as the indicator of how prepared students really are for the world that awaits them. It is imperative to examine our students against yet another standard, a standard that values both **content and process,** a standard that can reflect the depth of the students' mathematical understanding, a standard that is taught in the classroom every day and is documented as student performance.

The state level of ASAP consists of mathematics assessments which evaluate the progress of Arizona schools in preparing students to reason, apply mathematical concepts and communicate their thinking on a sample of Essential Skills.

The following are examples of the types of assessment items included in the ASAP mathematics assessments at the state level. These questions are from the eighth grade ASAP sample assessment from Riverside Publishing Company. This assessment takes place in an industrial arts classroom. The problems illustrate how fractions are multifaceted and assess the depth of the students' understanding. The students must be able to demonstrate a basic understanding of the relationships between fractional sizes, define the problem and compute to determine the results, illustrate a visual understanding of the problem, and explain their thinking.

In your industrial arts class you are making projects with wood. You and three of your friends—Jan, Keisha and Earl—decide to make birdhouses for your first project.

Exercise A

Your teacher, Mr. Ramirez, gives the four of you one piece of wood 64 inches long and gives each of you a slip of paper that reads,

"Measure the board and cut off 1/4 of it."

- First Jan measures the board and cuts off 1/4 of it. She gives the rest of the board to Keisha.
- Next, Keisha measures the board and cuts off 1/4 of it. She gives the rest of the board to Earl.
- Finally, Earl measures the board and cuts off 1/4 of it. He gives the rest of it to you.

You discover that you have much more wood than you need. Keisha and Earl complain that they don't have enough.

1. Draw a picture of the 64-inch board on the grid below and show where the students cut it. Show your calculations.

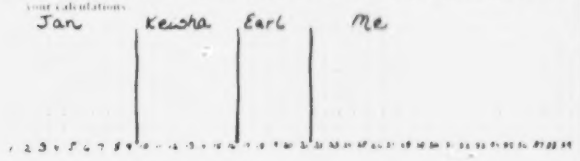
2. Explain what was confusing about Mr. Ramirez's directions and tell how he could have made them clearer.

BEST COPY AVAILABLE

The following are examples of student products from the eighth grade sample assessment. Examine them to see how each student attempted to solve the problems. What conclusions can be made about the students' understanding of the problems and the mathematical concepts needed to solve them?

MATHEMATICS

1. Draw a picture of the 64-inch board on the grid below and show where the students cut it. Show your calculations.



39 x 1/4 = 9.75 = Jan
30 x 1/4 = 7.5 = Keisha
23 x 1/4 = 5.75 = Earl

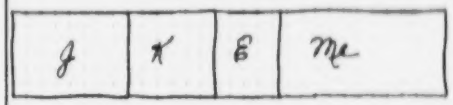
2. Explain what was confusing about Mr. Ramirez's directions and tell how he could have made them clearer. Use the Review Check List below to help make your answer complete.

What was confusing about Mr. Ramirez's directions was he said to measure and cut the board into 4. What he should have said was cut the board into fourths or divide the board into fourths and then cut it.

214

MATHEMATICS

1. Draw a picture of the 64-inch board on the grid below and show where the students cut it. Show your calculations.



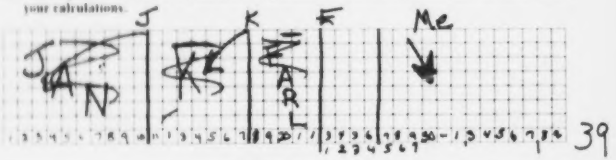
4√32 8
4√24 6
4√18 4.5
J = 8
K = 6
E = 4.5
M = 13.5

2. Explain what was confusing about Mr. Ramirez's directions and tell how he could have made them clearer. Use the Review Check List below to help make your answer complete.

Mr. Ramirez said to measured at off 1/4 of the boards but what Mr. Ramirez should have said was to measure the board into equal 1/4 pieces and cut them off so each person would have 8 inches of wood.

MATHEMATICS

1. Draw a picture of the 64-inch board on the grid below and show where the students cut it. Show your calculations.



Computation Box

2. Explain what was confusing about Mr. Ramirez's directions and tell how he could have made them clearer. Use the Review Check List below to help make your answer complete.

Mr. Ramirez said to each cut off 1/4 of it. He should've said to cut the board into fourths. One Forth for each of the 4 people. If they would've done this they wouldn't have had different sizes of wood.

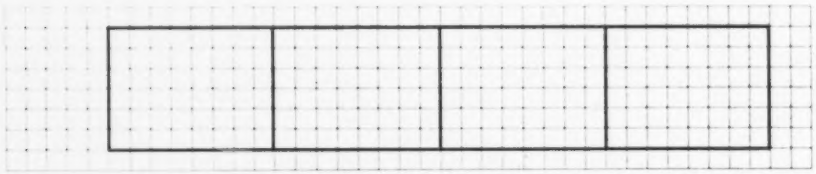
205

BEST COPY AVAILABLE

The students' responses are scored using a generic 4-point rubric, a standard set of criteria describing each score point. A headnote is included for each item in the assessment, describing the essential components and/or an example of one possible way to solve the problem. Both of these items are scored using the generic rubric.

Observation 1

Acceptable responses:
The diagram below shows an acceptable response. The width of the board may vary. The scale of the students' diagrams may be different.



The computation box should contain the following computations (or equivalents) or computations which correctly match the scale the students used in the diagram:

1/4 (64)=16

1/4 (64-16)=1/4 (48)=12

1/4 (48-12)=1/4 (36)=9

Observation 2

Acceptable responses: An acceptable response would address the following two parts of the problem:

What was confusing about the directions (i.e., The students did not realize that Mr. Ramirez meant 1/4 of the whole board.)?

How could Mr. Ramirez have made the directions clearer (i.e., He could have suggested that the students measure the board and divide it into fourths before doing any cutting.)?

Generic 4-point rubric

A 4 response represents an effective solution. It shows complete understanding of the problem, thoroughly addresses all points relevant to the solution, shows logical reasoning and valid conclusions, communicates effectively and clearly through writing and/or diagrams, and includes adequate and correct computations and/or set up.

A 3 response contains minor flaws. Although it shows an understanding of the problem, communicates adequately through writing and/or diagrams, and generally reaches reasonable conclusions, it shows minor flaws in reasoning and/or computation or neglects to address some aspect of the problem.

A 2 response shows gaps in understanding and/or execution. It shows one or some combination of the following flaws: an incomplete understanding of the problem, failure to address all aspects of the problem, faulty reasoning, weak conclusions, unclear communication in writing and/or diagrams, or a poor understanding of relevant mathematical procedures or concepts.

A 1 response shows some effort beyond restating the problem or copying given data. It shows some combination of the following flaws: little understanding of the problem, failure to address most aspects of the problem, major flaws in reasoning that lead to invalid conclusions, or a lack of understanding of relevant mathematical procedures or concepts.

Assign a 0 if the response shows no understanding of the problem or if the student fails to respond to the item.

Assign an NS (Not Scorable) if the response is illegible or written in a language other than English.

When scoring students' papers, you will find that some are clearly a 4 and some are clearly a 3, 2 or a 1. These papers are easy to score; but more times than not, there is not a clear cut delineation. For example, a paper will often have characteristics of both a 3 paper and a 2 paper. In this case, the decision for scoring becomes more complicated. This paper needs to be examined more carefully. Does the paper have more characteristics of a 3 paper or does it have more characteristics of a 2 paper? Few papers will have all the characteristics of a specific score point; but if the paper has more characteristics of one point than another, that is the score it should be assigned. Examine the students' products on the previous page. How would you evaluate and score each of them?

Using performance-based assessments, we can now begin to understand the depth of the students' mathematical ability and their ability to apply these concepts in a problem-solving context.

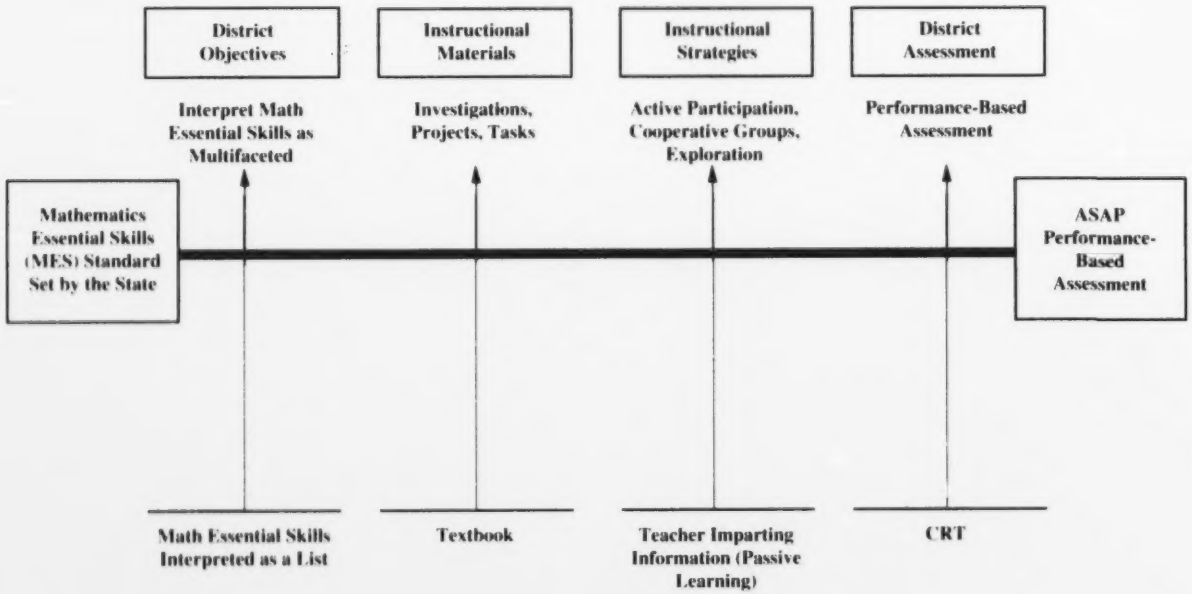
District Assessment Plan (DAP)

Districts are required by law to file a District Assessment Plan with the Arizona Department of Education. This plan establishes the district's expectations and the standards for student achievement. The DAP will have a significant impact on mathematics instruction. Districts are expected to build assessment programs that are aligned to the standards set by the *Arizona Essential Skills for Mathematics* in both content and process to the depth demonstrated by the ASAP mathematics assessments. Districts must decide what strategies they will use to determine students' progress in meeting the standards.

The DAP impacts more than just district assessment practices. Revising or creating assessment instruments also directly affects other components: district objectives, instructional materials and instructional strategies.

All four of these components are integral, and an effective plan cannot be developed without addressing all four simultaneously and keeping them in alignment. If students are to meet the standards set by districts, what is assessed should be aligned with what is taught; and instructional strategies utilized in the classroom should reinforce what is valued.

Curriculum Alignment Continuum

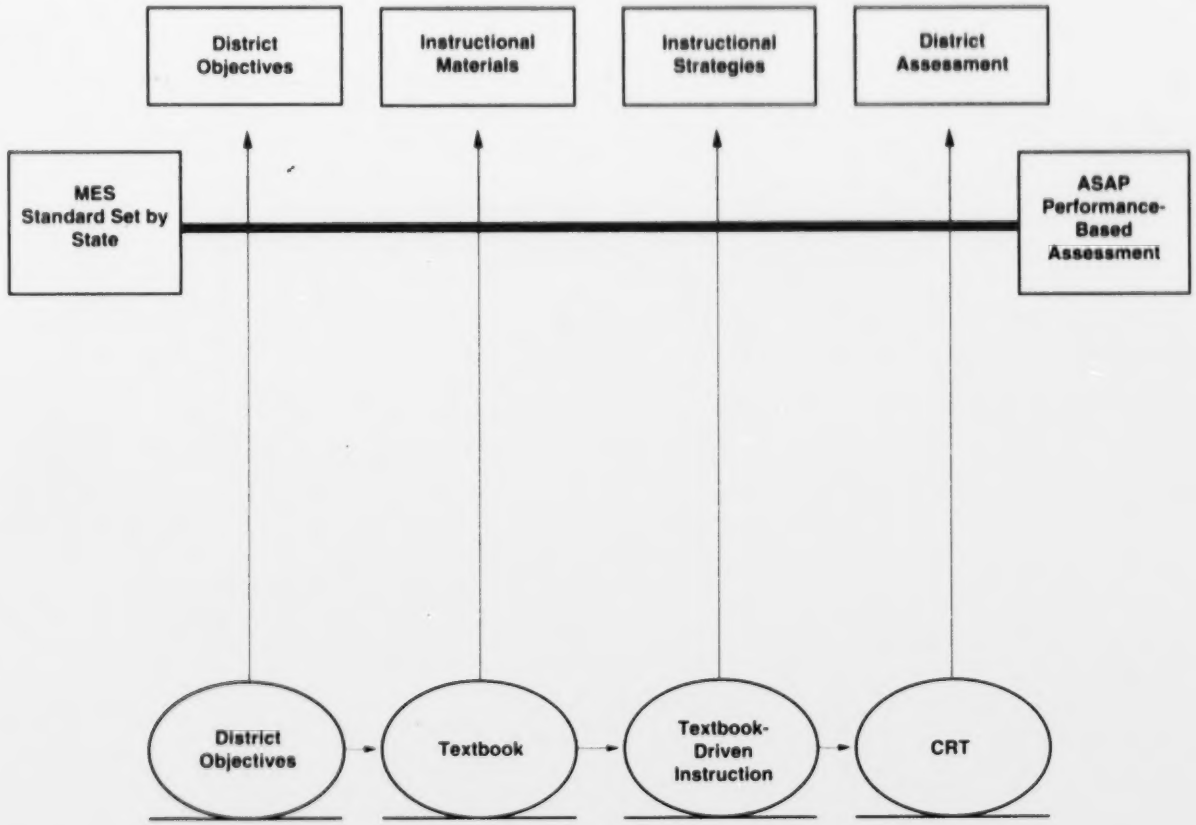


Many districts have already aligned all four components. However, districts must examine where their present system lies on the continuum and determine if it meets the standards set by the mathematics Essential Skills and the ASAP.

This district may have all four components aligned but sits on the bottom of the continuum. This plan does not assess students' progress in meeting the standards set by the state. It has interpreted the mathematics Essential Skills as a list, taught it accordingly and assessed students' progress in isolation.

Where does your district fit on the continuum? Will the placement enable your district to reach the standards set by the state?

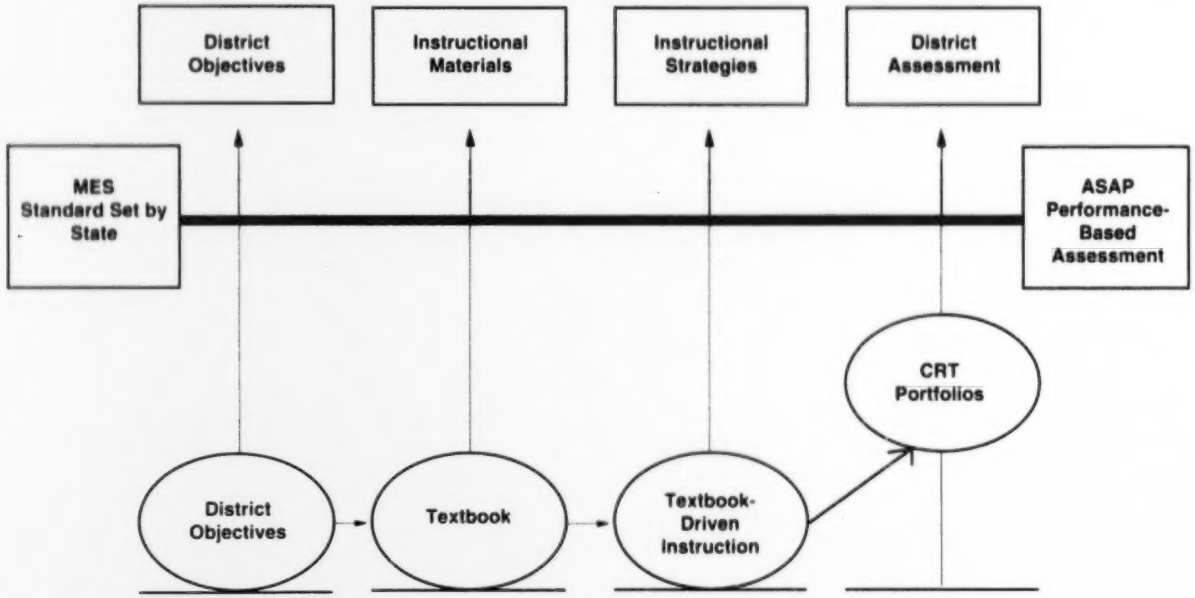
Curriculum Alignment Continuum



As districts make decisions about how they will assess their students, they must be cognizant of how their assessment practices interrelate with the other three components. If a district elects to use a combination of CRTs and portfolios, it raises district assessment up the continuum. There is no longer an alignment among all four components.

The use of portfolios calls for the collection of student work involving short tasks, projects, explanations and/or investigations. Unless changes are made to the remaining three components, inaccurate conclusions will be drawn by the districts. What is being assessed is not being taught.

Curriculum Alignment Continuum



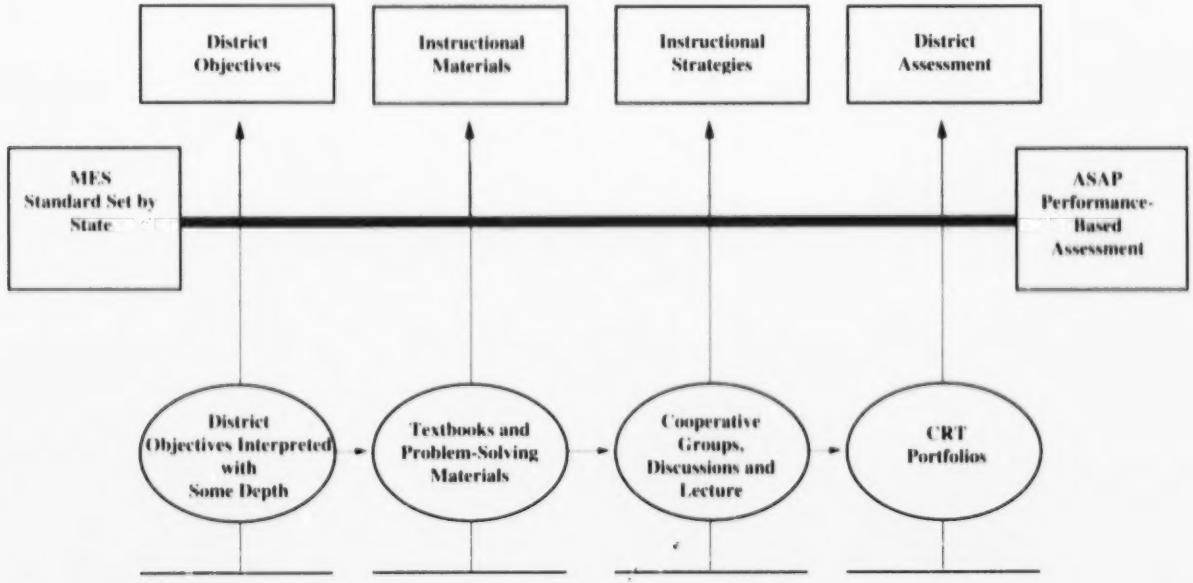
214

215

In order for students to meet the new standards set by the district, all the four components must be realigned. District objectives must be reinterpreted for depth, additional classroom materials must be added to provide experience with projects and investigations, and teachers must receive additional training to facilitate new types of learning experiences for the students in the classroom.

The District Assessment Plan can be a powerful instrument for changing mathematics from a static discipline to a dynamic process and empowering students mathematically.

Curriculum Alignment Continuum



216

217

Performance in the Classroom

Most mathematics educators have a dream of what students ideally should be experiencing in the classroom, e.g., working in small groups or independently; designing and doing investigations and projects; using tools such as manipulative materials, calculators, computers, assorted textbooks and other reference materials; consulting with each other and with the teacher; and keeping notebooks and other written reports of their progress, problems and potential solutions. The entire class may discuss a particular point of view or problem. The curriculum is rich in real problem solving and includes a full range of mathematical ideas. Mathematical power is at work in every corner of the classroom. Our vision of integrating the instructional process and assessment supports this dream.

The dream requires integrating the instructional process and assessment. The ASAP and DAP reinforce this integration and are designed to encourage classroom teachers to address mathematics from a problem-solving perspective. Problem solving is complex. It involves the recall of facts, the use of a variety of skills and procedures, the ability to evaluate one's own thinking and progress while solving the problem, and the ability to communicate one's thinking. Furthermore, success in problem solving also depends on the students' interest, motivation, and self-confidence.

The NCTM *Curriculum and Evaluation Standards* has an entire standard devoted to mathematical disposition. Mathematical disposition is the student's entire approach to solving a given task or problem. The *NCTM Standards* defines mathematical disposition as

- confidence in using mathematics
- flexibility in exploration
- willingness to persevere
- interest, curiosity, and inventiveness
- inclination to monitor and reflect on one's own performance
- valuing the application of mathematics
- appreciation of the role of mathematics in culture

Students' mathematical disposition then relates to how they position themselves when faced with a problem involving mathematics.

Instruction must now include the development and refinement of thinking skills such as understanding and formulating questions, conditions, variables, conjectures and methods of analyzing data. Student problem solvers need to be able to assess the reasonableness of their answers. This requires developing good estimation techniques. The inclusion of problem-solving strategies in instruction will strengthen students' confidence and help them develop positive attitudes and beliefs concerning their abilities. Students need to be taught how to use specific mathematical knowledge as well as when to use that knowledge. It is important for students to develop the skills that enable them to solve complex problems successfully. Assessment must parallel and be interwoven with that instruction. Evidence or indicators of student performance in each phase of problem solving become the assessment. Techniques vary from simple teacher observations to formal evaluations of student work using a scoring rubric.

There are several techniques for assessing the critical relationship between performance, attitudes and beliefs regarding problem solving:

- observing and questioning students
- evaluating student products
- using student self-assessments

Each of these techniques accesses information about the student through a form of communication. The key factor in any assessment is what the student communicates. Just as we recognize many different learning styles, there are also many different ways that students communicate what they do and do not understand. Thus, we have assessment techniques in which an impartial observer (the teacher) records observed actions and comments, and we use direct questioning to determine measured responses. The student has the opportunity to think through the processes of problem solving in writing student reports and in answering many of the questions in performance-based assessments. Since each of these techniques addresses different areas, it is logical to use a variety of techniques with each student. In preparing students for performance-based assessment, teachers need to structure experiences that teach students to analyze their own thinking processes. Students must begin to understand why those skills are needed, reshape attitudes and develop habits that continue the use of those critical thinking skills.

220

In the world of work, people are valued for the tasks or projects they do well, their ability to work with others and their response to problem situations. To prepare students for future success, both curriculum and assessment must promote this kind of performance.

As we move into performance-based assessment, we move out of the traditional "grading" system. Student work looks different from skill-oriented assignments. The intent is now to draw out student thinking processes. For example, the following papers are the result of an assignment given to several different classes where students were asked to count the number of people in their class and then determine how many handshakes there would be if each student were to shake hands once with every other student.

The problem was posed verbally by the teacher; and the students were asked to write the question in their own words, work out a solution, show their work and explain the strategies they used. Examine the student products on the following page. What do the responses say about the students' mathematical disposition and their ability to communicate their understanding?

221

There are thirty-seven people in our classroom. If everyone shakes hands, how many handshakes?

today we

36	403	591
+35	+22	+10
91	425	601
+34	+21	+9
105	446	610
+33	+20	+8
138	466	618
+32	+19	+7
170	485	625
+31	+18	+6
201	503	631
+30	+17	+5
231	520	636
+29	+16	+4
260	536	630
+28	+15	+3
288	551	633
+27	+14	+2
315	565	635
+26	+13	+1
341	578	636
+25	+12	
366	580	
+24	+11	
390	591	
+23		
403		

there are 636 handshakes

I got this answer by drawing 37 lines and then I would get the first line and make it shake hands and so on and when it was done shaking hands I would erase it.

Homework:

There are 37 people in our classroom. If everyone shakes hands, how many hand shakes?

[Remember! Yesterday we found a way to write this as the sum of consecutive numbers. (ex 1 up to 36) Today we used the staircase to build a rectangle to find the sum of consecutive numbers.]

Solution
1+2=3
3+4=7
5+6=11
7+8=15
9+10=19
11+12=23
13+14=27
15+16=31
17+18=35
19+20=39
21+22=43
23+24=47
25+26=51
27+28=55
29+30=59
31+32=63
33+34=67
35+36=71
666

answer 666

There are 21 people in our classroom If everyone shakes hands, how many handshakes?

20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
+1
210

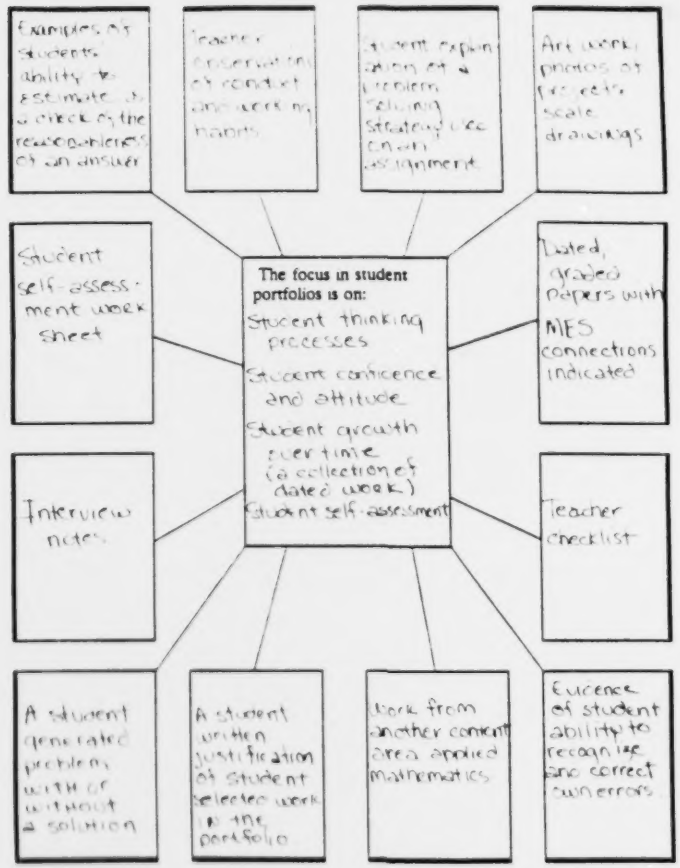
There are 210 handshakes

One person goes down the line and shakes everyones hand. That person would shake 20 peoples hand. The next person would go and he would shake one less persons hand and that would go one decreasing by one everytime). You would end up with 210 handshakes

Using an assessment portfolio will enable teachers to assess student progress more easily. Portfolios give the teachers the opportunity to examine how students are performing individually and/or in cooperative groups, using both formal and informal means of assessment. Portfolios allow teachers to analyze the students' level of understanding and ability to apply mathematical concepts in context. Many teachers are accustomed to maintaining student folders, housing their work for a specific period of time. The portfolio differs from the folder in that it becomes a permanent record of examples of student work that specifically demonstrate skill and performance capabilities. A student portfolio will also contain written teacher observations and student self-analysis. The portfolio is a historical record which documents the students' growth, skill development and mathematical disposition as learners.

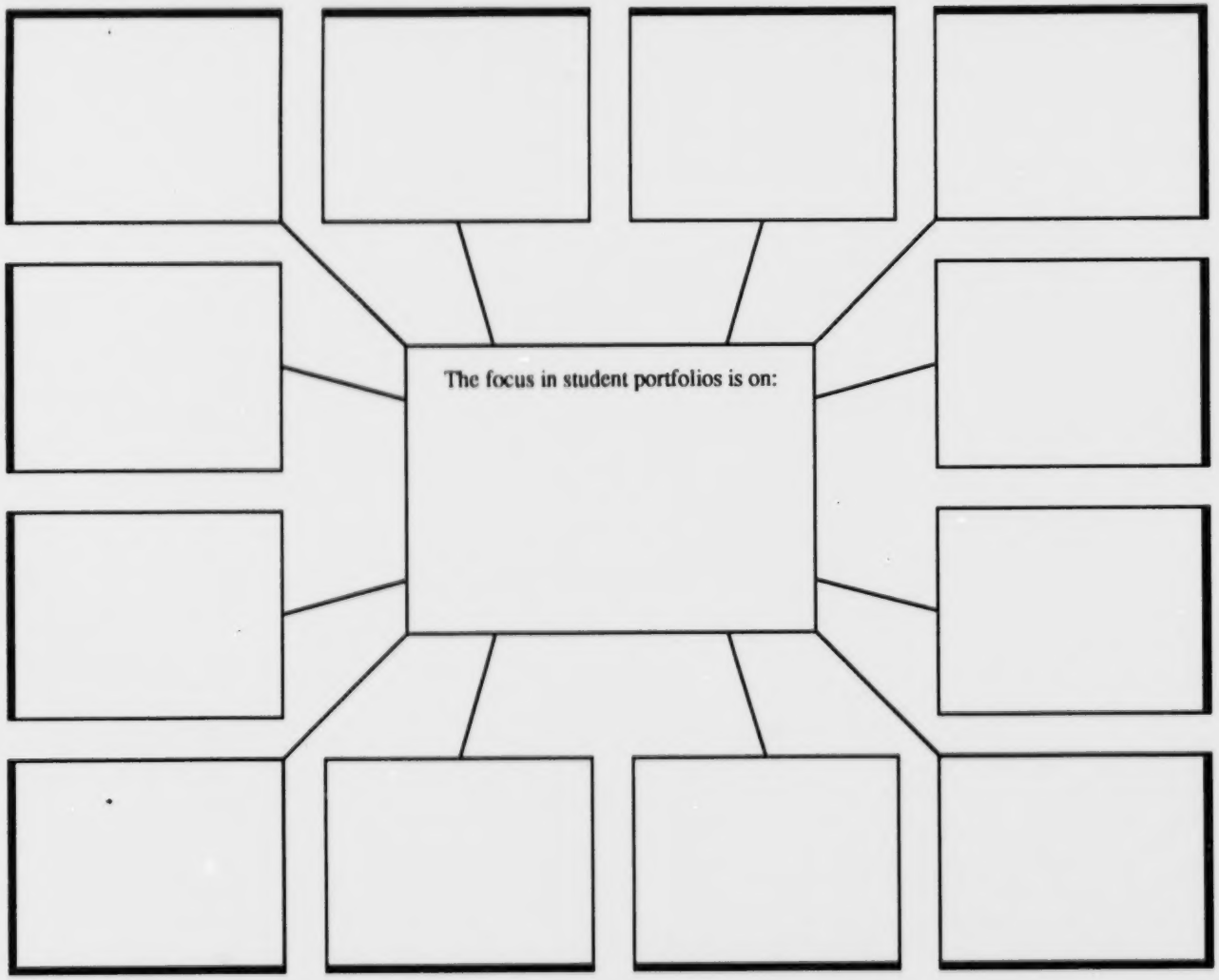
The focus of an assessment portfolio must be determined by the student and the teacher using district guidelines. It should measure the progress the students are making in reaching the standards set by the district. A generic rubric could be used to assess the students' progress against the standard. The following example lists items which might be included in an assessment portfolio. A portfolio needs to have a clear focus and direction in order to determine clearly students' mathematical disposition and empowerment.

MAPPING A PORTFOLIO


Examples of student's ability to estimate as a check of the reasonableness of an answer
Teacher observation of conduct and working habits
Student explanation of a problem solving strategy used on an assignment
Art work: photos of projects, scale drawings
Student self-assessment work sheet
The focus in student portfolios is on:
Student thinking processes
Student confidence and attitude
Student growth over time (a collection of dated work)
Student self-assessment
Dated, graded papers with NJES connections indicated
Interview notes
Teacher checklist
A student generated problem with or without a solution
A student written justification of student selected work in the portfolio
Work from another content area applied mathematics
Evidence of student ability to recognize and correct own errors

A blank portfolio map is provided for your use.

MAPPING A PORTFOLIO*



*From Far West Laboratory

Scoring

The mathematics assessments for ASAP are now using a generic rubric for scoring. This rubric sets the standards and expectations for Arizona students. It is used to score all of the ASAP mathematics assessments. The generic rubric was adopted to give educators more opportunities for giving students credit for unusual and unique responses that are mathematically sound.

Since this rubric is generic, its versatility goes beyond the ASAP assessments. It sets a standard and an expectation for students' work. It can be used to assess mathematical tasks, projects, investigations, portfolios and/or district objectives.

Generic 4-Point Rubric

A 4 response represents an effective solution. It shows complete understanding of the problem, thoroughly addresses all points relevant to the solution, shows logical reasoning and valid conclusions, communicates effectively and clearly through writing and/or diagrams, and includes adequate and correct computations and/or set up.

228

A 3 response contains minor flaws. Although it shows an understanding of the problem, communicates adequately through writing and/or diagrams, and generally reaches reasonable conclusions, it shows minor flaws in reasoning and/or computation or neglects to address some aspect of the problem.

A 2 response shows gaps in understanding and/or execution. It shows one or some combination of the following flaws: an incomplete understanding of the problem, failure to address all aspects of the problem, faulty reasoning, weak conclusions, unclear communication in writing and/or diagrams, or a poor understanding of relevant mathematical procedures or concepts.

A 1 response shows some effort beyond restating the problem or copying given data. It shows some combination of the following flaws: little understanding of the problem, failure to address most aspects of the problem, major flaws in reasoning that lead to invalid conclusions, or a lack of understanding of relevant mathematical procedures or concepts.

Assign a 0 if the response shows no understanding of the problem or if the student fails to respond to the item.

229

In reviewing this rubric, it becomes obvious that curriculum and instruction will need to look different. We must analyze the rubric carefully and see how it compares to the components defined under mathematical power, e.g., mathematical thinking, mathematical understanding, tools and techniques, and communication skills. We must develop guidelines to help our students meet the new standards and show them how this translates into student performance.

Mathematical Thinking: Students must first use their knowledge and understanding to analyze the problem. They should use inductive and deductive reasoning to evaluate and interpret the major issues, make conjectures and develop a strategy that will yield an effective solution to the problem.

Mathematical Understanding: Students must apply their understanding of mathematical concepts and procedures in solving the problem, be able to explain the processes used and determine the significance of the data they have obtained.

Tools and Techniques: Students often need to be able to demonstrate their mathematical understanding in different ways, ranging from modeling conceptual understanding through the use of hands-on materials to a high level of application using computers and/or calculators in both instructional and assessment settings.

Communication Skills: Communicating mathematically is a crucial part of mathematical empowerment. An explanation of what the results are, what they mean and how the students know, is as important as the process itself. Effectively communicating the solution to a problem enables others to understand exactly what the students have done, the depth of their understanding and their ability to make mathematical connections. Responses can range from verbal or written explanations; diagrams, tables or graphs; numerical representations of the problem; and/or a combination of any of the above. The ability to communicate mathematically is a necessary and powerful tool.

Using these guidelines, we can begin to move our students closer to mathematical empowerment by helping them become thinking individuals who can analyze, interpret, conjecture, explain and draw logical conclusions.

Resources

California Department of Education. (1991). *A Sampler of Mathematics Assessment*. Sacramento: Office of State Printing.

Carnegie Commission on Science, Technology, and Government. (September 1991). *In the National Interest: The Federal Government in the Reform of K-12 Math and Science Education*. New York: The Carnegie Commission.

Far West Laboratory (1992) Portfolios: *A Collaborative Assessment System*. San Francisco, CA: Far West Laboratory for Educational Research and Development.

Governor's Office, State of Arizona (1991). *Governor's Task Force on Educational Reform*. (audiovideo cassette)

Leitzel, James R. C. (ed.) (1991). *A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics*. Washington, D.C.: The Mathematical Association of America.

Lewis, Anne (1989). *Restructuring America's Schools*. Arlington, VA: American Association of School Administrators.

Mathematical Sciences Education Board (1991a). *Counting on You: Actions Supporting Mathematics Teaching Standards*. Washington, D.C.: National Academy Press.

Mathematical Sciences Education Board (1991b). *For Good Measure: Principles and Goals for Mathematical Assessment*. Washington, D.C.: National Academy Press.

Mathematical Sciences Education Board (1989). *Mathematics Education: Wellspring of U. S. Industrial Strength*. Washington, D.C.: National Academy Press.

National Center for Education Statistics (June 1991). *The STATE of Mathematics Achievement: NAEP's 1990 Assessment of the Nation and the Trial Assessment of the State*. Washington, D.C.: U.S. Department of Education.

National Council of Teachers of Mathematics (March 1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

National Council of Teachers of Mathematics (March 1991). *Professional Standards for Teaching Mathematics*. Reston, VA: The National Council of Teachers of Mathematics, Inc.

National Research Council (1991). *Moving beyond Myths: Revitalizing Undergraduate Mathematics*. Washington, D.C.: National Academy Press.

National Research Council (1990a). *Renewing U.S. Mathematics: A Plan for the 1990s*. Washington, D.C.: National Academy Press.

National Research Council (1990b). *Reshaping School Mathematics: A Philosophy and Framework for Curriculum*. Washington, D.C.: National Academy Press.

National Research Council (1989). *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*. Washington, D.C.: National Academy Press.

Riverside Publishing Company (1992). *Mathematics Sample Assessment*. Chicago, IL.: Riverside Publishing Co.

Steen, Lynn Arthur (ed.) (1990). *On the Shoulders of Giants: New Approaches to Numeracy*. Washington, D.C.: National Academy Press.

Stenmark, Jean Kerr (1989). *Assessment Alternatives in Mathematics: An overview of assessment techniques that promote learning*. Berkeley, CA: California Mathematics Council and EQUALS, 1989.

Trafton, Paul R. (ed.) (1989). *New Directions for Elementary School Mathematics*. (1989 NCTM Yearbook). Reston, VA: The National Council of Teachers of Mathematics, Inc.

U. S. Department of Education (1991). *America 2000: An Education Strategy*. Washington, D. C.: U. S. Department of Education.

Willoughby, Stephen S. (1990). *Mathematics Education for a Changing World*. Alexandria, VA: Association for Supervision and Curriculum Development.